OULU BUSINESS SCHOOL

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BETTING AGAINS BETA WITH CONDITIONAL MODELING IN BELGIUM STOCK MARKET

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UNIVERSITY OF OULU ABSTRACT OF THE MASTER'S THESIS

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Abstract

Background and objectives

CAPM implies that there should exists positive relation between the returns' and betas' of the stocks and this relation should be equal to size of market risk premium. However empirical research has found that this relationship is too flat or even completely flat. Low beta stocks perform better and high beta stocks perform worse, than expected according their beta. To measure performance difference there has been created betting against beta (BAB) factor, which goes long to low beta stocks and shorts high beta stocks and positions are levered to have neutral market position. It has been shown that this factor generates positive four factor risk adjusted returns in USA market and in numerous international markets. BAB factors risk adjusted returns have been one the highest in Belgium stock market.

This study checks can unconditional or conditional asset pricing models with time varying betas explain returns related to BAB factor in Belgium stock market. It is also investigated how the factor exposures of BAB factor vary over time and different market conditions. Further it is checked can investor achieve statistically significant alpha in five year time interval by tilting his portfolio towards BAB factor in Belgium stock market.

Data and methodology

It is used daily and monthly factor return data from Belgium stock market from July 1990 to December 2015. Data is achieved from AQR Capital Management's database. BAB returns are investigated through basic static factor regression models. To capture time variance on the factor exposure there is generated conditional factor models for BAB returns using smoothed Kalman filter. Also BAB returns varying exposures and alphas are checked through rolling regression with 5 year time window.

Results

Unconditional or conditional regression models can't explain returns of BAB factor in Belgium stock market. In all broad sample regressions there exists statistically significant alpha. Adding momentum to three factor model cuts down alpha, but still statistically significant part of the returns are unexplained by the four factor model. In bear market times alpha related to BAB factor decreases substantially. BAB factor has negative market exposure most of the time. Overall BAB has positive exposure to momentum factor, which goes extremely strong in bear market times, but in bull markets this exposure vanishes. Overall BAB factor's factor exposures get stronger in bear market times, except with size factor. Rolling regressions show that investor can rarely achieve statistically significant risk adjusted returns with 5 year investment horizon by tilting his portfolio towards BAB factor. Tilting portfolio towards BAB factor neither penalizes the investor in the unique way in the bad times.

Keywords

BAB, Betting against beta, Factor, Time vary, Conditional, Kalman filter, Smoothing, Bear market

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1 INTRODUCTION

1.1 Background and motivation

Throughout history of existing stock markets investors have been keen to predict the returns of assets and generate methods which could provide them bigger returns anomalously or through exposure to risk factors. In the beginning of stock markets there where no way predict returns, existed only chaos. First change to unpredictability was capital asset pricing model. Model suggested that assets with low respond to changes in aggregate market return should have low returns and those that respond highly to aggregate changes should have high returns. This was the first step to cross sectional predictability, predictability between the assets. Reasoning behind this is that assets with high response are riskier than assets with low response, because when economy state is bad and aggregate market goes down high responsive assets lose more of their value. For the investors returns in those bad times are more valuable, because aggregate consumption in those times is lower. This because in bad times they lose their job more often, interest rates are higher and the aggregate investment returns are lower. So every dollar they receive in bad times gives them higher satisfaction than in good times. To compensate this high sensitivity with states of economy, high responsive assets should provide higher total returns estimated over all states of economy.

Assets sensitivity to market movements in called market beta or just beta. It is affected by variation of the assets returns, variation of aggregate market returns and the correlation between these two. So it is unique for every asset. To measure beta it is used historical data of the returns of the asset. With longer time periods it reasonable to think that assets characteristics change and it makes betas change. Usually beta is estimated with one year interval.

There is huge variation between the assets and also the characteristics of specific assets. Also asset prices are highly volatile. For these reasons academic research concentrates on investigating portfolios' returns instead of separate assets. Portfolios are compiled from individual assets by sorting them with some specific criteria. Portfolios returns should be affected by certain states of economy with same way for longer time periods than individual assets. Portfolios are more stable in this sense, but it is reasonable to think that also portfolios characteristics in the way how economy states affect them are time varying. With portfolios it is possible to control certain variables affect to the return so that portfolio's returns rise from effect of the certain variable without other variables biasing that too much.

After the capital asset pricing was introduced it was found out that its explanatory power was limited. Aggregate market returns and assets sensitivity to it couldn't explain all returns. There was need for other kind of variables that capture the states of economy and generate positive return in the long run, same way as does market returns. These variables are called risk factors and market excess return over risk free rate is one of them. Others well known factors are size factor, value factor, Momentum factor and liquidity factor. There are many other factors, but those above are most robust and widely documented. There is debate that should betting against beta to be included as one of the risk factors. This factor takes into account possible risk factor related to low market beta stocks. Positive return of betting against beta factor indicates that low market beta stocks provide higher market risk adjusted returns compared to high market beta stocks.

Classical asset pricing models predict constant betas for risk factors. These models are called unconditional. There is lot evidence that factor betas fluctuate over time. That is very reasonable sight because whole structure of economy changes in longer time periods. For these reasons there have been created conditional asset pricing models where betas are time varying.

This thesis main idea is the explore betting against beta factor in Belgium stock market. It has been documented that in Belgium's betting against beta factors risk adjusted returns are one of the highest in the world. It is examined how well traditional risk factors can explain those returns. Also it is checked are those BAB factor's returns better explained if we use conditional asset pricing models with time varying betas. Time varying betas are constructed with random walk model of Kalman filter and smoothing. It is also explored with rolling regression can investor higher risk adjusted returns by tilting his portfolio towards betting against beta factor.

1.2 Research questions and hypotheses

The research problems are

1) Can unconditional or conditional regression models with classical dynamic risk factors, market, size, value and momentum explain returns related to low beta factor, BAB, in Belgium stock market?
2) What kind of factor exposures are related to BAB and how exposures vary over time and market conditions?
3) Can investors achieve positive statistically significant four factor risk corrected returns in modest time interval by tilting their portfolio to towards BAB factor?

It is hypothesized that other risk factors cannot explain BAB factor in static models with monthly data for returns. Not even when BAB is regressed on all the other factors, market MKT, size SMB, value HML and momentum UMD. This has been true with monthly data in earlier research with different time period concerning data collection (Frazzini & Pedersen 2014). Concerning about daily data for returns it is hypothesized also that other factors cannot predict BAB returns. It is achieved more data points compared to monthly data so standard error for estimated alpha will go relatively smaller. This sense it is logical that if monthly returns cannot be explained neither can be daily returns. It will be interesting to see can three factor model perform much better than one factor model and can four factor model contribute much compared to three factor model

It is expected that conditional regression explains returns better than unconditional one with same regressors. It is hard to predict do conditional regression models with less regressors perform better than unconditional regression with more regressors. Conditionality should improve model, but also adding factors should do the same. It is impossible to say beforehand which one has larger effect. It would be highly surprising if conditional CAPM could explain BAB related returns in such way that there would not exists any statistically significant alpha. Predicting how well BAB returns are explained with conditional multifactor models is hard.

It is expected that there is huge time variation on factor exposures of BAB factor. Time variation of betas has documented in previous studies with individual stocks and different kind of portfolios, which are constructed based on some factor. BAB factor's exposures to other factors are expected to reach higher levels in bear markets, because in generally assets carrying risk premium tend to correlate more in recessions.

It is expected that with shorter time periods which replicate real life investors horizon significant abnormal returns related to BAB factor disappear. Confidence intervals for alpha expand with shorter time periods and it is more likely that positive alpha can rises from pure randomness.

This thesis proceeds as follows. Chapter 2 captures earlier researches evidence concerning over performance concerning low beta and volatility stocks. There is also introduced possible rational and behavioral explanations concerning this phenomenon. Demand and success of conditional asset pricing models is introduced. In chapter 3 it is introduced data and statistical and mathematical background, factor construction procedures, regression equations and Kalman filter and smoothing methods. Chapters 4 shows empirical results concerning betting against beta factor in Belgium stock market. Factor performance is evaluated through static and time varying regression models. BAB factors time varying exposure to other factors is presented. Chapter 5 concludes the main findings.

2 BETA AND VOLATILITY ANOMALY WITH TIME VARYING MODELING

2.1 Beta anomaly

Capital asset pricing model (CAPM) states (Sharpe 1964) that exposure to market risk should be linearly compensated by higher excess return over the risk free rate and the ratio between return and exposure should equal market risk premium. Measure to exposure of market risk is called beta. Beta is a measure of systematic and undiversifiable risk. If asset returns would follow CAPM perfectly returns of assets would set in thin line described in figure 1.

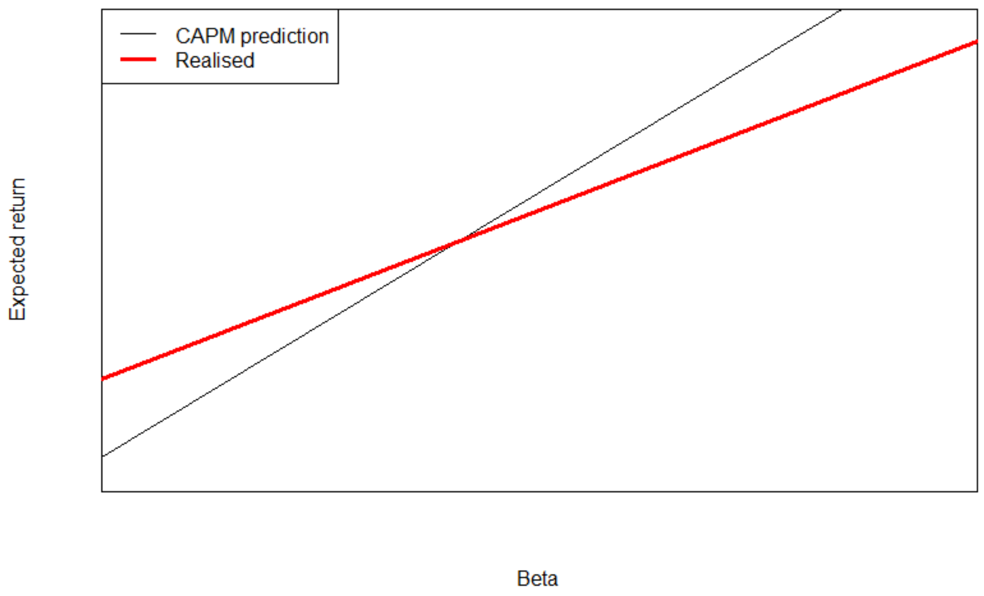


Figure 1. CAPM prediction and realized returns. Figure displays difference between the CAPM predicted return and actual realized return

However there exists lot evidence that relationship between beta and risk premium is too flat compared to CAMP prediction. Higher beta stocks do not provide as high returns as they should based on their market beta and low beta stocks provide too high returns compared to their beta. High beta portfolios have generated negative alpha and low beta portfolios positive alpha, when those performance has been evaluated by the CAPM. Alpha being part of the returns which couldn't been explained by the model.

This tendency is described with thick line in the figure 1. (Adrian & Franzoni 2009, Ang, Chen et al. 2006, Friend & Blume 1970, Jensen et al. 1972.)

In the deriving the CAPM there made are several assumptions. (1) All investors have common idea of joint probability distributions of returns of all available assets. (2) Probability distributions of returns of available assets are jointly normal. (3) Investors choose asset weights so that they maximize their end of period value of the portfolio and all investors are risk averse. (4) Investor can take long and short positions of any size in any asset, also in risk free asset. Investors may borrow and lend with riskless rate of interest. Assumption (2) to hold time period should be infinitesimal. With finite period returns are log normal. Otherwise (2) and (3) three are considered to hold with high enough accuracy. (Black 1972.) Violations considering assumption (1) have been shown to have very little effect on CAPM (Lintner 1969). There exists though severe restrictions related to assumption (4). Restrictions in short-sell mechanisms can lead to low beta stocks to overcome high beta stocks. Cash received from short-sales cannot be immediately used make other transactions. Also entering short positions demands cash collaterals. Short selling is general is more hard than going simply long. (Blume & Friend 1973.) This together with big differentiation between investors' estimations concerning high beta stocks future returns causes that high beta stocks prices are more determined by the overconfident optimists (Baker et al. 2011). Yet it is possible that CAPM may be robust to violations of frictionless short selling mechanism if optimal holdings of each investor would not consist any short positions. This being true all investors portfolio would consist linear combination of market portfolio and zero beta portfolio like risk free asset as government bonds.(Blume & Friend 1973.)

More prominent explanation for low beta related excess returns, which CAPM couldn't explain, are the borrowing constrains related to assumption (4). There is explored two separated cases. One assumes that there is no riskless asset and no riskless borrowing or lending is possible. Other case is that there exists riskless asset and long positions are possible, but short positions are not. In both cases short selling of risky asset can be done frictionless. It has been shown that slope for return to beta line, like in figure 1, is smaller when there exits either kind of restrictions concerning the borrowing compared to situation when there is no restrictions.(Black 1972.) Individual and institutional investors suffer from the borrowing constrains and their chase of higher returns leads them overweight the riskier high beta stocks instead of using the leverage and just investing to market portfolio. This causing high beta stock prices increase and returns to decrease. (Frazzini & Pedersen 2014.)

CAPM has not been able to explain also other kind of portfolios, which are arranged based on size, book-to-market or momentum variables (Carhart 1997, Fama & French 1992). The clear trend in those portfolios alpha, has lead of creation of risk factors concerning size and book-to-market number, which is also called value, and momentum. Those factor portfolios are called SMB, HML and UMD. Because those factor portfolios generate positive mean returns in longer time intervals despite they have equity neutral market positon, but suffer negative returns in bad states of the economy, they have been interpret as sources of undiversifiable risk which is not captured by the market risk. (Fama & French 1992, Fama & French 1993.)

Low beta stocks abnormally high returns reflected to camp have been explained through institutional asset management industry tendencies. Often asset managers are benchmarked against return of the market portfolio. Manager benefits for placing assets to high beta stocks, if they just give little higher returns, being at the same time indifferent concerning the additional risk. For example portfolio manager can tilt towards higher beta stocks instead of market portfolio so that portfolios beta increases 10% and same time returns increase 5%. If manager is benchmarked against just raw returns of market portfolio without risk adjustment manager will benefit from increase in raw returns, when actually risk adjusted returns are decreased. (Baker et al. 2011.) Decentralized and inefficient two-step investment process of asset management companies is one explanation. There unsophisticated investment committee makes first asset class allocation decision. Capital is allocated to different asset managers who pick up the assets in different asset classes. Unsophisticated committee tends to allocate money to asset classes that with above average performance. This may lead to situation where for the asset manager it is more important to outperform in up than down markets and this causing high beta stocks to come overcrowded.(Blitz & Van Vliet 2007.) Also other behavioral explanations have been given. Over all investors are volatile averse, but concerning lottery type gambles things chance. In gambles where probability of small loss is big and probability of big win is small utility value of the gamble is highly overstated against actual expected value in money. So people are more willing to risk when payoffs are positively skewed and with little risk there exists very slight chance of huge payoff, which could have big impact to one's wealth. Many times high beta stocks offer these kinds of payoffs. Most of the time their prices decline, but rarely prices can double or triple. Representative bias can also cause low beta anomaly. When looking extremely successful companies, the high volatility or high beta seems to be common factor in the beginning of their history. High beta being representative character for becoming great success. This approach, by concentrating only on biggest winners, forgets all those high beta companies that lost their value and even ended to bankruptcy. (Baker et al. 2011.)

To measure beta anomaly or capture possible risk factor related to low beta stocks there was constructed betting against beta factor (BAB). This factor goes long to low beta stocks and take short position high beta stocks. Long and short position are levered to have beta of one. This causing total market position to be risk neutral in the sense of CAPM. (Frazzini & Pedersen 2014.)

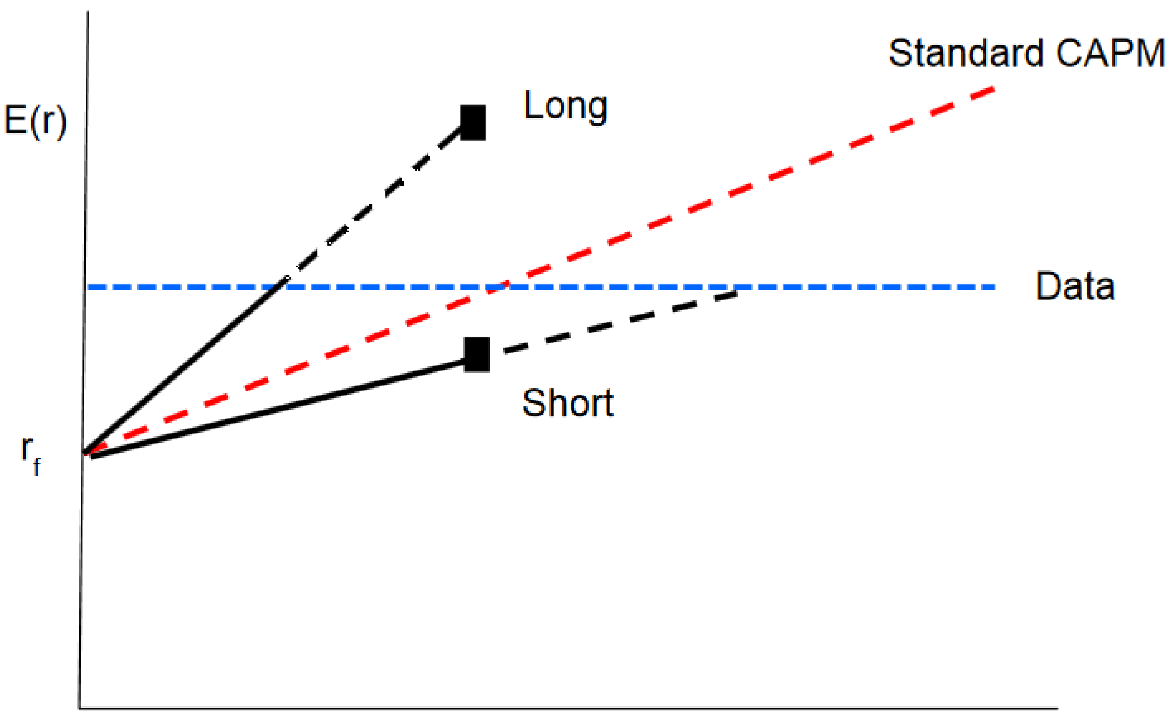


Figure 2. BAB factor construction. High beta stocks are delevered and low beta stocks are levered to have beta of 1 . (Ang 2014)

There have been found statistically significant unexplained returns for BAB factor in USA even after risk adjustments to market, size, value, momentum and liquidity with 1-factor, 3-factor, 4-factor and 5-factor pricing models. BAB factor survived internationally from 4-factor pricing model in six of 19 countries, by generating statistically significant alpha. Alpha being one of the strongest in Belgium. It is shown that during tightening liquidity constrains more money is flown to high beta stocks this causing BAB factor realize negative returns and at the same time expected returns to rise. (Frazzini & Pedersen 2014.)

2.2 Low volatility anomaly

Closely related to beta anomaly is volatility anomaly. Expected returns according the traditional pricing models are determined by how returns covary with risk factors. Idiosyncratic volatility is volatility that is independent of risk factors. Idiosyncratic volatility is volatility of error term of the pricing model. Idiosyncratic volatility can be diversified away and should not have affect to expected returns, under assumption of perfect pricing model. Models are always simplified descriptions of reality and do not describe world perfectly and if idiosyncratic volatility has effect on returns the relationship should be positive. Investors that are willing to carry idiosyncratic risk should be paid for that, causing relationship if there is to be positive. Empirical facts anyhow show total opposite. Past idiosyncratic and aggregate volatility are negatively related to returns (Ang, Hodrick et al. 2006), even after controlling numerous other factors. Across 23 developed markets difference of top and bottom quintile portfolios monthly returns, sorted based on passed idiosyncratic volatility, has been negative and statistically significant, even after controlling world market, size and value factors. This phenomenon being statistically significant individually for every G7 country. (Ang et al. 2009.) Also concerning contemporaneous idiosyncratic and aggregate volatility there has been found same kind of relationship (Ang 2014). Explanations for idiosyncratic volatility to be negatively related to returns are same kind as for low beta anomaly. It has been found that lottery preferences and market frictions explain 29-54% of idiosyncratic volatility puzzle in individual stock level and 78-84% in volatility sorted portfolio level (Hou & Loh 2016). Low volatility anomaly seems to be even stronger than beta anomaly. Bottom quintile portfolio structured based on volatility has beaten top quintile portfolio with over 1000% in total return on time interval 1968-

2008 for all CRSP stocks. Same phenomenon exists even if it used only 1000 biggest stocks of CRSP based on their market capitalization. Bottom quintile portfolio of volatility beating top quintile portfolio with over 600% in same time interval. Sorting to volatility portfolios was done based on estimation period volatility, but stocks that had low volatility in the estimation period also had lower volatility in the actual performance period where the returns were measured. This making low volatility puzzle even stronger. Also constituents in the high volatility portfolios were more varying this making transaction costs higher for them and difference even actually even bigger. Low returns of high volatile stocks could be rationalized by they being less risky in sense that they would provide higher returns in most severe economic downturns like 1973-1974, 2000-2002 and in financial crises 2008. But this is not the case. High volatile stocks lose even more than aggregate market in those catastrophic events. (Baker et al. 2011.)

There has also contrarian evidence that there exists no statistically significant relation between the idiosyncratic volatility and expected returns or at least the relation is sample specific. It is found that when using NYSE stocks instead of CRSP stocks quintile portfolio construction phenomenon diminishes substantially. Also equal weighting instead of value weighting in counting portfolio returns vanishes the negative cross-sectional relation between the idiosyncratic volatility and returns. By constructing quintile portfolios of idiosyncratic volatility so that each portfolio contains equal amount of market capitalization instead of equal number of shares phenomenon disappears completely.(Bali & Cakici 2008.)

2.3 Time varying asset pricing models

Ever more growing number of risk factors and unconditional pricing models incapability explain returns has raised demand for conditional pricing models, which could predict returns more precisely or with less factors. Unconditional pricing models rely conventional ordinary least squares method (OLS) in estimating factor loadings. OLS procedure assumes asymptotical standard errors, constant factor loadings and stable variation in risk factors. All these assumptions are not in line with the actual data. (Ang & Chen 2007.) It is very reasonable to think that individual firms' and portfolios' relative risk for the cash flows varies over time and business cycles. Firms in bad shape may need to increase their leverage compared to other firms in recessions and this causes their market betas to rise. Also technology shocks affect to different sectors fluctuate and relative share of different sectors of whole economy differ. All this causing betas to variate over time. It is also problematic that unconditional CAPM relies the fact that stock market return is reliable measure for aggregate wealth return. (Jagannathan & Wang 1996.)

It has been shown that in time period 1926-1975 most of CRSP stocks and portfolios constructed of have significant variation on their market beta, when returns are predicted with conditional CAPM. This variation has been detected under assumption that betas follow random walk proses. This variation is strongly inherent in sub periods of original timeframe. (Sunder 1980.) Unconditional CAPM for CRPS stocks (Fama & French 2006) has created higher betas for high book value stocks then for growth stocks in 1926-1963, but opposite is true in later sample 1964-2001, which causes high book value stocks generate abnormal returns valuated by CAPM in later period. Conditional market betas for portfolio that goes long to top decile and short in lowest decile on stocks by their book-to-market value has varied between 3 in late 1930s and -0,5 in end of 2001. Previous are examples of long time evolution of market beta, but there has been documented also short interval variation. Variation in market betas is violation against OLS assumptions. It causes that CAPM cannot be used to assess the fit for conditional CAPM. Betas and risk premium are also correlated, which makes OLS to provide biased estimates for alpha and beta compered to conditional alpha and beta. It is not surprising that when market betas and risk premium are correlated and conditional alpha is zero, there arises subsamples where unconditional alpha statistically differs from zero. This was the case with value stocks in CRPS data in 1964-2001.(Ang & Chen 2007.) It has been suggested (Avramov & Chordia 2006) that well known anomalies or risk factors like size, book-to-market, momentum and other firm characteristics in cross sectional returns actually rise from the dynamic behavior of market beta. Turning into conditional CAPM explained return spreads between book-to-market sorted portfolios in time 1926-2001 in USA stock market. With conditional CAPM, where betas are time varying, marker risk premium is predictable and volatility being stochastically systematic there exists no sign of conditional alpha for book-to-market strategy. (Ang & Chen 2007.)

One natural extension in turning from unconditional CAPM to conditional one is taking into account asymmetric sensitivity to market movements between by specifying separately upside beta and downside beta. It is shown that investors are loss averse and place greater weight in their utility functions on same size loss then same size gain. Loss aversion leads that stocks with high sensitivity market downward movements, having great downside beta, generate overall higher returns. It is approximated that down side beta risk premium between extreme quintile portfolios is 6% per annum even with controlling other known cross-sectional factors like size, value, momentum and liquidity. Downside beta premium is different from the coskewness effect. Past downside beta is good predictor for becoming downside beta for becoming month and returns with longer time interval for most of the stocks. (Ang et al. 2006.)

Conditional asset pricing modeling with conditional betas could be done with instrumental variables, but it has been shown that (Harvey 2001) estimates of betas are sensitive to choose of instrumental variables which are choose to proxy the time variation in betas. There is evidence (Avramov & Chordia 2006) that conditional betas, which are approximated from macroeconomic variables and firm characteristics can approve asset pricing models for individuals stocks and diminish size and book-to-market factors, but still leaving the momentum and liquidity unexplained. Another way of achieving time varying betas is treat them as latent variables. Latent variable betas are estimated from past time series of themselves. To estimate time varying betas there has been used autoregressive model with lag of one AR(1) (Ang & Chen 2007), but mean reverting process with Kalman filter has been more successful (Adrian & Franzoni 2009). Process is rationalized by that investors are unaware of becoming level of risk and they try to estimate this risk with current level of beta and its long run historical mean. In the conditional CAPM process with Kalman filter expected beta rises from continuing learning process. With conditional CAPM using Kalman filter pricing errors were smaller for portfolios sorted by size and B/M then with standard conditional CAPM where betas rise from state variables. (Adrian & Franzoni 2009.) It has also been found that stochastic mean reverting process for market beta, like Kalman filter, generates more precise approximation than GARCH modelled beta (Brooks et al. 1998), beta conditioned with firm level variables or rolling regression of betas (Ang et al. 2006). Rolling regression betas also strongly misspecify true systematic risk when beta shocks are strongly persistent. Conditional CAPM using Kalman filter provides better explanation to classical pricing anomalies like size, value and idiosyncratic volatility than other conditional models. (Ang et al. 2006, Jostova & Philipov 2005.)

3 DATA AND METHODS

3.1 Data and factor creation

Data is collected from AQR Investing database. There are used factor mimicking portfolios for low beta -, market -, value -, size - and momentum factor. These factor mimicking portfolios are called BAB, MKT, HML, SMB and UMD. Data is collected from beginning of July 1990 to end of December 2015. All dynamic portfolios BAB, HML, SMB and UMD are long/short portfolios, with zero cost, so that total net position becomes zero and market neutral. There are calculated daily and monthly simple returns for all these portfolios. Returns are calculated in US dollars. For HML and SMB portfolios rebalancing concerning constituents happens in June in every calendar year. For BAB and UMD portfolios constituents rebalancing happens monthly. MKT, HML, SMB and UMD portfolios are value weighted by their market capitalization and weights are rebalanced every month. BAB portfolio weights are also rebalanced every month, but weights are based on their past market beta. Lowest beta stocks have higher weights in the long position and highest beta stocks have higher weights in short position. Specific details concerning this procedure follow later.

Matrices and vectors are bolded to different them from scalars. Vectors are vertical unless specified otherwise. Matrix and vector transposes are indicated with apostrophe.

Return for MKT portfolio return is market return over the risk free rate. It is generated from market capitalization value weighted long position in all available stocks in Belgium stock market and short position in one-month US treasury bill.

$$\mathrm{MKT} = r_{\mathrm{market}} - r_{treasury} \tag{1}$$

For the SMB, HML and UMD factor creation stocks are sorted. Based on market value of equity stocks are sorted in two size portfolios with breakpoint being 80th percentile. Based on stocks' book-to-market valuation stocks are sorted three value portfolios and three momentum portfolios based on stocks recent 12 month performance excluding previous month. Breakpoints for both of these sorts being 30th and 70th percentile. To get balanced factors, which are not disturbed by other factor effects, it is done double sorting. Sorting is done conditionally, so that first sorting is done with size factor and then with other factor. This differs from Fama and French method (Fama & French 1992), who use independent sorts. With conditionally sorting it is ensured balanced number of securities in each portfolio. In independent sorting all stocks are separately sorted to by size and book-to-market ratio. This can lead situation where for example big value-portfolio consists very little stocks if there exists small number of big stocks with high book-to-market ratio.

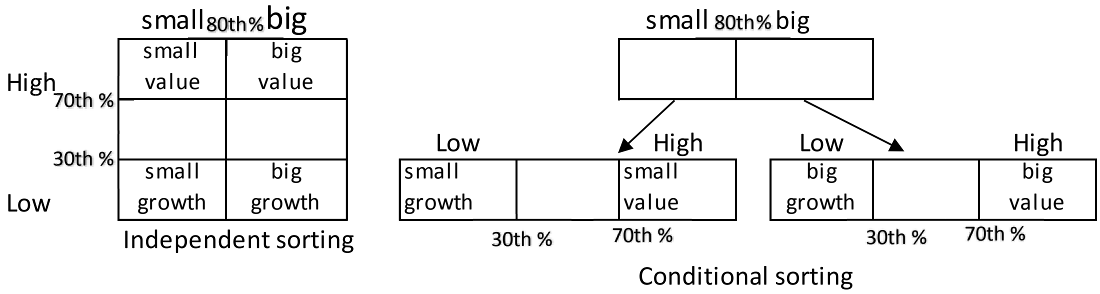


Figure 3. Independent sorting vs. Conditional sorting for HML portfolio. By conditional sorting it is securized balanced number of shares in every portfolio

SMB factor is constructed based on size and value sorts. SMB return is average of three small portfolios minus average of three big portfolios.

$$\begin{aligned} SMB = & \ 1/3\,(\text{Small Value} + \text{Small Neutral} + \text{Small Growth}) \\ & - 1/3\,(\text{Big Value} + \text{Big Neutral} + \text{Big Growth}) \end{aligned} \tag{2}$$

HML factor is constructed based on size and value sorts. HML return is average of two value portfolios minus average of two growth portfolios.

$$\begin{aligned}\text{HML} &= 1/2\,(\text{Small Value} + \text{Big Value}) \\ &- 1/2(\text{Small Growth} + \text{Big Growth})\end{aligned} \tag{3}$$

UMD factor is generated from size and momentum sorts. UMD return is average of two high performed portfolios minus average of two low performed portfolios.

$$\text{UMD} = 1/2\,(\text{Small High} + \text{Big High}) - 1/2(\text{Small Low} + \text{Big Low}) \tag{4}$$

BAB factor goes long for low-beta stocks and short on high-beta stocks. Those long and short positions are scaled by their estimated betas to achieve beta of one for both positions. This leading total position to have beta of zero, being market neutral. BAB factor return is calculates as follows

$$BAB_{t+1} = \frac{1}{\beta_t^L}(r_{t+1}^L - r^f) - \frac{1}{\beta_t^H}(r_{t+1}^H - r^f) \tag{5}$$

Here β_t^L and β_t^H are estimated betas of low and high beta portfolio from estimation period, r_{t+1}^L and r_{t+1}^H are returns of those portfolios from actual period.

To form long and short positions each stock is ranked on basis of their beta from estimation period. Beta is achieved from rolling regressions of daily excess returns on daily value-weighted market excess returns. Estimated time series beta for stock i is

$$\hat{\beta}_i^{ts} = \frac{\hat{\rho}\hat{\sigma}_t}{\hat{\sigma}_m} \tag{6}$$

, where $\widehat{\sigma_l}$ and $\widehat{\sigma}_m$ are estimated volatilities for stock and market returns and $\hat{\rho}$ is correlation between the returns. For volatilities there are used 1 year rolling standard deviation and 5 year horizon for correlation. This is because correlations seems to be more stable than volatilities (Santis & Gerard 1997). To estimate volatilities there are used 1-day log returns and overlapping 3-day log returns log returns to control non-synchronous trading. It is required at least 120 trading days of non-missing data to estimate volatilities and at least 750 trading days of non-missing data for correlations. To get final estimation period betas $\hat{\beta}_i$ time series betas $\hat{\beta}_i^{ts}$ are shrinked towards cross-sectional mean $\hat{\beta}$.

$$\hat{\beta}_i = \gamma\hat{\beta}_i^{ts} + (1-\gamma)\hat{\beta} \tag{7}$$

Cross sectional mean is settled $\hat{\beta}$=1 and shrinking factor γ = 0,6. This shrinking will not affect to rankings of stocks based on beta, but will affect to scaling beta parameter in equation 5.

Stocks weights in the portfolios are based on their rank on $\hat{\beta}_i$. Let $z_i = rank(\hat{\beta}_i)$ and $\boldsymbol{z}$ to be $n \times 1$ vector of those ranks, n being number of stocks and ranking to be ascending related to beta. Average rank is then $\bar{z} = \mathbf{1}_n'\boldsymbol{z}/n$, where $\mathbf{1}_n'$ is $1 \times n$ vector of ones. Weights for assets in low beta and high beta portfolios are

$$\mathbf{w}_\mathrm{L} = k(\bar{\mathbf{z}} - \mathbf{z}_\mathrm{a}), \mathbf{w}_\mathrm{H} = k(\mathbf{z}_\mathrm{b} - \bar{\mathbf{z}}) \tag{8}$$

, here $\bar{\mathbf{z}}$ is constant vector of $\bar{z}$'s with as many elements as $\boldsymbol{z_a}$ or $\boldsymbol{z_b}$. For counting $\boldsymbol{w_L}$ and $\boldsymbol{w_H}$ is used $\boldsymbol{z}$ vectors ranks above and below $\bar{z}$, $\boldsymbol{z_a}$ and $\boldsymbol{z_b}$. k is normalizing constant $k = \frac{2}{\mathbf{1}'|\boldsymbol{z}-\bar{\mathbf{z}}|}$ and $\bar{\mathbf{z}}$ is constant vector of $\bar{z}'s$ with n elements.

Low beta returns in equation 5 are received from $r_{t+1}^{L} = \boldsymbol{r_{t+1}^{a\prime}}\boldsymbol{w_L}$, where return vector $\boldsymbol{r_{t+1}^{a}}$ is related to those stocks which beta rank is above the average. Similarly for high beta $r_{t+1}^{H} = \boldsymbol{r_{t+1}^{b\prime}}\boldsymbol{w_H}$, here return vector is related $\boldsymbol{r_{t+1}^{b\prime}}$ stocks, which beta rank is below the average. Scaling betas are calculated with same logic $\beta_t^L = \boldsymbol{\beta_t^{a\prime}}\boldsymbol{w_L}$ and $\beta_t^H = \boldsymbol{\beta_t^{b\prime}}\boldsymbol{w_H}$. BAB factor generation procedure follows Frazzini and Pedersen method (Frazzini & Pedersen 2014).

3.2 Methods

3.2.1 Unconditional modeling

It is examined can positive returns related to BAB factor be explained by CAPM, Fama French three factor model or four factor model with momentum added to three factor model. First this problem is attacked with unconditional regression with constant. BAB factor is regressed on different risk factors.

$$BAB_t = \alpha + \boldsymbol{\beta}'\mathbf{X}_t + \varepsilon_t \tag{9}$$

Here α is part of BAB return which other risk factors are unable to explain. $\boldsymbol{\beta}$ is vector, which represents factor loadings for different risk factors. It is not expected that error terms are independently and identically distributed. Instead it is used Newey-West procedure (Newey & West 1986) for error terms to get heteroscedasticity and autocorrelation corrected standard deviations. It is examined with student's t-test that are alphas and betas on different regression statistically significant.

3.2.2 Conditional modeling

It also checked that can conditional factor models explain BAB returns. Conditionality is generated through smoothed Kalman filter. It is expected that betas follow random walk procedure around their previous observation. Beta estimates are smoothed under assumption that both return series for BAB and other factors X are known. It is generated conditional model without intercept. General model for BAB return in time step t is.

$$\begin{aligned} BAB_t &= \boldsymbol{\beta}_{\mathbf{t}}{}'\mathbf{X}_{\mathbf{t}} + \varepsilon_t \\ \beta_t^i &= \beta_{t-1}^i + w^i \end{aligned} \tag{10}$$

Error terms of regression and beta processes are expected to follow $\varepsilon_t \sim N(0,E)$ and $w_t^i \sim \mathrm{N}(0,W^i)$. Indexation i refers to specific column in $\boldsymbol{\beta}$, so it specifies certain factor's loadings. $\boldsymbol{\varepsilon}$ vector is expected to be uncorrelated with each $\boldsymbol{\beta}^{\mathbf{i}}$ vector , which is necessary condition for applying Kalman filter. This procedure is run separately for CAPM, Fama-French three factor model and Cahart four factor model. For place of explaining variable $\mathbf{X}_{\mathbf{t}}$ it is used specific factor returns. Model is generated without intercept, because if there was generated regression with intercept by smoothened Kalman filter, that would have arose the problem that intercept's / alpha's significance could not be estimated.

To proceed Kalman filtering and smoothing it is needed to proceed maximum likelihood estimation for the variance of the BAB return E and covariance matrices of betas $\mathbf{W}$. By denoting $\boldsymbol{\psi}$ the vector of parameters $\{E,\mathbf{W}\}$, $\boldsymbol{\psi}$ can be estimated by maximum likelihood using prediction error decomposition of log-likelihood

$$\hat{\boldsymbol{\psi}}_{MLE} = \arg\max_{\boldsymbol{\psi}} \ln L(\boldsymbol{\psi} \mid \mathbf{BAB}_T) = \sum_{t=1}^{T} \ln f(BAB_t \mid \mathbf{BAB}_{\mathbf{t-1}}; \boldsymbol{\psi}) \tag{11}$$

Here $\mathbf{BAB}_T$ refers the vector of whole return series of BAB factor and $\mathbf{BAB}_{\mathbf{t-1}}$ refers the return series until time step t-1.

Kalman filter is set of recursion equations for determining optimal estimates for betas with given information until time t. Filter is generated from two sets of equations, prediction equations and updating equations.

To describe the Kalman filter, let the optimal estimator vector of betas in certain time step when past return series **BAB**$_t$ is known, to be $\hat{\boldsymbol{\beta}}_{\mathbf{t}} = E[\boldsymbol{\beta}_{\mathbf{t}} \mid \mathbf{BAB}_{\mathbf{t}}]$. Let the covariance matrix of $\hat{\boldsymbol{\beta}}_{\mathbf{t}}$ to be $\hat{\mathbf{C}}_{\mathbf{t}} = E\left[(\boldsymbol{\beta}_{\mathbf{t}} - \hat{\boldsymbol{\beta}}_{\mathbf{t}})(\boldsymbol{\beta}_{\mathbf{t}} - \hat{\boldsymbol{\beta}}_{\mathbf{t}})' \mid \mathbf{BAB}_{\mathbf{t}}\right]$.

In prediction equation phase let the $\hat{\boldsymbol{\beta}}_{\mathbf{t-1}}$ and $\hat{\mathbf{C}}_{\mathbf{t-1}}$ be known at time t-1. Optimal predictors for $\boldsymbol{\beta}_{\mathbf{t}}$ and $\mathbf{C}_{\mathbf{t}}$ are

$$\begin{aligned}
\hat{\boldsymbol{\beta}}_{\mathbf{t|t-1}} &= E[\boldsymbol{\beta}_{\mathbf{t}} \mid \mathbf{BAB}_{\mathbf{t-1}}] = \hat{\boldsymbol{\beta}}_{\mathbf{t-1}} \\
\hat{\mathbf{C}}_{\mathbf{t|t-1}} &= E\left[(\boldsymbol{\beta}_{\mathbf{t}} - \hat{\boldsymbol{\beta}}_{\mathbf{t-1}})(\boldsymbol{\beta}_{\mathbf{t}} - \hat{\boldsymbol{\beta}}_{\mathbf{t-1}})' \mid \mathbf{BAB}_{\mathbf{t-1}}\right] = \hat{\mathbf{C}}_{\mathbf{t-1}} + \mathbf{W}
\end{aligned} \tag{12}$$

The corresponding optimal predictor of BAB$_t$ with given information at t-1 is

$$\hat{BAB}_{t|t-1} = E[BAB_t \mid \mathbf{BAB}_{\mathbf{t-1}}] = \hat{\boldsymbol{\beta}}'_{t|t-1}\,\mathbf{X}_{\mathbf{t}} \tag{13}$$

Prediction error e_t and its predicted covariance $\hat{Q}_t$ are

$$\begin{aligned}
e_t &= BAB_t - \hat{BAB}_{t|t-1} = BAB_t - \hat{\boldsymbol{\beta}}'_{\mathbf{t|t-1}}\mathbf{X}_{\mathbf{t}} = (\boldsymbol{\beta}_{\mathbf{t}} - \hat{\boldsymbol{\beta}}_{\mathbf{t|t-1}})'\,\mathbf{X}_{\mathbf{t}} + \varepsilon_t \\
\hat{Q}_t &= E[e_t e_t'] = \mathbf{X}'_{\mathbf{t}}\,\hat{\mathbf{C}}_{\mathbf{t|t-1}}\,\mathbf{X}_t + E
\end{aligned} \tag{14}$$

In updating phase, when new observations BAB$_t$ comes available optimal predictions $\hat{\boldsymbol{\beta}}_{\mathbf{t|t-1}}$ and $\hat{\mathbf{C}}_{\mathbf{t}|t-1}$ are updated by using

$$
\begin{aligned}
\hat{\boldsymbol{\beta}}_t &= \hat{\boldsymbol{\beta}}_{t|\mathbf{t-1}} + \hat{\mathbf{C}}_{\mathbf{t|t-1}}\,\mathbf{X_t}\,\hat{Q}_t^{-1}\,e_t \\
\hat{\mathbf{C}}_{\mathbf{t}} &= \hat{\mathbf{C}}_{\mathbf{t|t-1}} - \hat{\mathbf{C}}_{\mathbf{t|t-1}}\,\mathbf{X}_t\,\hat{Q}_t^{-1}\,\mathbf{X}_{\mathbf{t}}'\,\hat{\mathbf{C}}_{\mathbf{t|t-1}}
\end{aligned}
\tag{15}
$$

By Kalman smoothing it is achieved final estimates for betas. For smoothing whole return series for all factors are expected to be known and it is proceeded backwards in the time series of the returns and filtered estimates to achieve smoothened estimates for each beta $\hat{\beta}_t^i$. Once all whole return series $\mathbf{BAB_T}$ is observed, the optimal estimates of betas $\hat{\boldsymbol{\beta}}_{t|T}$ and its covariance matrix $\hat{\mathbf{C}}_{t|T}$ can be computed from

$$
\begin{aligned}
\hat{\boldsymbol{\beta}}_{\mathbf{t|T}} &= E\left[\boldsymbol{\beta}_{\mathbf{t}} \mid \mathbf{BAB}_T\right] = \hat{\boldsymbol{\beta}}_{\mathbf{t}} + \hat{\mathbf{C}}_{\mathbf{t}}^{*}(\hat{\boldsymbol{\beta}}_{\mathbf{t+1|T}} - \hat{\boldsymbol{\beta}}_{\mathbf{t}}) \\
\hat{\mathbf{C}}_{\mathbf{t}|T} &= E\left[(\boldsymbol{\beta}_{\mathbf{t}} - \hat{\boldsymbol{\beta}}_{\mathbf{t}|T})(\boldsymbol{\beta}_{\mathbf{t}} - \hat{\boldsymbol{\beta}}_{\mathbf{t}|T})' \mid \mathbf{BAB}_T\right] = \hat{\mathbf{C}}_{\mathbf{t}} + \hat{\mathbf{C}}_{\mathbf{t}}^{*}(\hat{\mathbf{C}}_{\mathbf{t+1|T}} - \hat{\mathbf{C}}_{\mathbf{t+1|t}})\hat{\mathbf{C}}_{\mathbf{t}}^{*\prime}
\end{aligned}
\tag{16}
$$

Here $\hat{\mathbf{C}}_{\mathbf{t}}^{*} = \hat{\mathbf{C}}_{\mathbf{t}}\hat{\mathbf{C}}_{\mathbf{t+1}|t}^{-1}$. The algorithm starts by setting $\hat{\boldsymbol{\beta}}_{\mathbf{T|T}} = \hat{\boldsymbol{\beta}}_{\mathbf{T}}$ and $\hat{\mathbf{C}}_{\mathbf{T|T}} = \hat{\mathbf{C}}_{\mathbf{T}}$ and then algorithm proceeds backwards for $t = T-1,...,1$.

Evaluating posterior variances $\hat{\mathbf{C}}_{\mathbf{t}|T}$ using recursive Kalman smoothing as above can lead numerical instability, which can cause covariance matrices to be nonsymmetrical and even negatively definite (Petris et al. 2009.). To get more robust estimates for $\hat{\mathbf{C}}_{\mathbf{t}|T}$ it is used sequentially updating singular value decomposition. Details concerning this algorithm can be found in (Oshman & Bar-Itzhack 1986).

Confidence intervals of these estimated betas for each time step separately can be calculated. Kalman smoothing together with singular value decomposition of $\hat{\mathbf{C}}_{\mathbf{t}|T}$ gives distribution of each factor beta in each time step, when return series of all factors is known $\beta^i_{t|T} \sim \mathrm{N}(\hat{\beta}^i_{t|T}, {\sigma^i_{t|T}}^2)$. In diagonal of $\hat{\mathbf{C}}_{\mathbf{t}|T}$ lies variances ${\sigma^i_{t|T}}^2$. Here also betas time varying pairwise covariance estimates $\mathrm{cov}(\beta^i_{t|T}, \beta^j_{t|T})$ are known. Estimating statistically significance of betas in whole time interval is meaningless, because by definition betas follow random walk procedure and for every time step there exists unique distribution for each beta.

Conditional alphas in each time step are estimated as follows.

$$\hat{\alpha}_t = BAB_t - \hat{\boldsymbol{\beta}}'_{\mathbf{t|T}} \mathbf{X_t} \tag{17}$$

where $\hat{\boldsymbol{\beta}}_{\mathbf{t}|T}$ is predicted vector of betas for time step t, when information set from whole time interval is known.

Final estimation for mispricing term $\hat{\alpha}$ is mean of the time series $\hat{\alpha}_t$ and standard error of the mean is estimated from same time series. This alpha estimation procedure follows (Adrian & Franzoni 2009).

3.2.3 Rolling regression

It is used 5 year rolling window to regress BAB factor on other factors. This way it is estimated can real life investor achieve statistically significant higher returns with modest time interval by tilting his portfolio towards BAB factor, so that those returns cannot be obtained through other factor exposures. Also rolling regressions imply about time evolution of the factor betas. BAB returns are regressed as in equation 17. It is applied Newey-West procedure to get corrected standard deviations.

$$BAB_t = \alpha_t + \boldsymbol{\beta}'_t \mathbf{X}_t + \varepsilon_t \tag{18}$$

To get time specific estimates for α_t and $\boldsymbol{\beta}_t$ it used previous 1305 (5 years, 261 market days in a year) daily observations of BAB factor and explaining factors $(BAB_{t-1,} \mathbf{X_{t-1}})$, $(BAB_{t-2,} \mathbf{X}_{t-2})$,…and $(BAB_{t-1305,} \mathbf{X}_{t-1305})$ to generate regression where those estimates are received.

4 DATA ANALYSIS

4.1. Summary statistics

Table 1 reports summary statistics related to BAB factor raw returns. Mean returns of BAB are positive and highly significant in broad sample. Standard deviations for calculating of sharp ratios and t-values of the mean, are Newey-West corrected to take into account heteroscedasticity and autocorrelation. BAB daily returns are positively skewed and skewness is highly significant. Extreme positive returns occur more often than corresponding negative returns. With other factors skewness is either negative and significant or slightly positive and not significant. This can be seen from summary statistics of other factors, which are presented in appendix 1. On the bull markets BAB returns are less skewed. Excess kurtosis of BAB is highly significant, implying fat tails of distribution. This meaning that extreme observations occur more often than it would be expected by normal distribution. Also kurtosis drops down in in bull markets. In daily basis kurtosis of BAB factor is much larger than other factors, but in monthly basis it is second smallest after HML.

Table 1. BAB factor summary statistics, July 1990 to December 2015

The table reports means, skewnesses and kurtosis of the BAB returns. In parentheses are t-statistics. T-values for means and sharp ratios are calculated by using Newey-West correction for standard deviation, except with monthly bear markets there is not enough observations to calculate Newey-West weights. Instead it is used Andrews weights to get corrected standard deviations. Statistically significant values, which p-value is lower than 5% are highlighted.

	Daily					Monthly				
Case	Means	Sharp	Skew.	Kurtos.	n	Means	Sharp	Skew.	Kurtos.	n
Broad	**0,041**	0,611	**3,205**	**72,128**	6654	**0,713**	0,609	**0,37**	**1,846**	306
	(2,92)		(106,75)	(1234,68)		(2,96)		(2,66)	(6,71)	
Bear	0,049	0,449	**2,682**	**31,639**	573	-0,250	-0,151	0,873	**3,269**	26
	(0,59)		(26,28)	(155,19)		(-0,23)		(1,93)	(3,93)	
Bull	0,024	0,383	0,114	**10,850**	1992	0,722	0,660	0,247	0,856	91
	(0,92)		(2,08)	(98,97)		(1,77)		(0,979)	(1,83)	

Table 2 reports correlations of factors daily returns in broad, bear and bull sample. Table 3 reports same correlations for monthly returns. It is used Spearman’s correlation to capture possible nonlinearities between the factors returns’ correlation. BAB returns are positively and statistically significantly correlated with UMD returns in broad samples. With daily and monthly return data correlation with BAB factor and MKT factor is negative, but statistically significant only with daily data. There exists positive significant correlation between the BAB and size factor in daily basis in broad – and both subsamples, but in monthly basis correlations are insignificant and signs differentiating between samples.

Table 2. Daily factor correlations, July 1990 to December 2015

Table reports Spearman's correlations between the factors using daily data. Bull (Bear) markets are times when trailing 12-month return has been over (under) 20% (-20%).Statistically significant correlations with significance level 5% are highlighted. Sample size are 6654, 573 and 1992 for broad, bear and bull sample.

	MKT			SMB			HML			UMD		
	broad	bear	bull	broad	bear	bull	broad	bear	bull	broad	bear	bull
BAB												
broad	**-0,29**			**0,13**			0,01			**0,13**		
bear		-0,51			**0,09**			0,04			**0,50**	
bull			**-0,20**			**0,16**			-0,04			**-0,06**
MKT												
broad				**-0,28**			**-0,03**			**-0,12**		
bear					**-0,19**			0,05			**-0,61**	
bull						**-0,38**			0,02			**0,16**
SMB												
broad							**-0,10**			**0,05**		
bear								-0,02			**0,10**	
bull									**-0,14**			**-0,06**
HML												
broad										0,00		
bear											0,01	
bull												**0,06**

On bull markets and with both data frequencies BAB negative correlation with MKT is lower than in broad sample. BAB correlation with SMB by daily returns is higher level in bull markets than in broad sample. With monthly returns negative correlation between BAB and SMB is decreased when moving from broad sample to bull sample.

BAB correlation with MKT is more negative in bear markets than overall with both data frequencies. BAB correlation with UMD is decreased substantially in bull markets and increased in daily basis in bear markets. This trend can be seen in both data frequencies. Though sample sizes for monthly factor returns of bull and bear market times are so small that correlations are not statistically significant. MKT factor's correlation with UMD and BAB has same kind of characteristics. There is negative correlation in broad sample and it gets more negative in bear markets. BAB correlation with HML is very small with both frequencies in broad sample, bull – and bear markets.

Table 3. Monthly factor correlations, July 1990 to December 2015

Table reports Spearman's correlations between the factors using monthly data. Bull (Bear) markets are times when trailing 12-month return has been over (under) 20% (-20%).Statistically significant correlations with significance level 5% are highlighted. Sample size are 306, 26 and 91 for broad, bear and bull sample.

	MKT			SMB			HML			UMD		
	broad	bear	bull	broad	bear	bull	broad	bear	bull	broad	bear	bull
BAB												
broad	-0,04			-0,08			0,02			**0,15**		
bear		-0,26			-0,06			0,09			0,28	
bull			0,00			-0,02			-0,07			0,06
MKT												
broad				0,00			0,06			**-0,19**		
bear					0,21			0,08			**-0,76**	
bull						**-0,21**			**0,22**			0,04
SMB												
broad							**-0,14**			**-0,18**		
bear								**0,18**			**-0,41**	
bull									**-0,09**			-0,04
HML												
broad										0,05		
bear											0,08	
bull												0,04

In both data frequencies from broad sample to bull sample it can be seen trend that correlation between BAB and HML factors turns negative and absolute value increases.

4.2. Unconditional regressions

In table 4 it is presented OLS regressions as an equation 9. Here Belgium stock market BAB factor returns are regressed on CAPM, Fama-French three factor model and Carhart four factor model. Standard errors of intercept and coefficient estimates are Newey-West corrected. CAPM, Fama-French three factor model or four factor model cannot capture returns related to BAB factor, not on daily basis or monthly basis. There exists statistically significant alpha in all explaining models. By comparing tables I and IV is can be seen that alpha in CAPM and the three factor model with daily basis is even higher than the raw return of BAB factor. Same is true for CAPM on monthly basis.

Table 4. Unconditional factor regressions for BAB factor, July 1990 to December 2015

Table reports OLS regression coefficients of CAPM, Fama-French three factor model and four factor model with momentum added. For 4-factor model there is separated regressions for bull and bear market. Bull (Bear) markets are times when past 12-month return has been over (under) 20% (-20%). Regressions are generated with daily and monthly data. In parentheses there appears t-statistics of coefficients. T-statistics are calculated with Newey-West corrected standard deviations. For Newey-West correction with daily data of broad sample there is used lag length of 9 and for monthly data of broad sample it is used lag length of 5. With daily data of bull market there is used lag length 7 and for bear market lag length 5. For monthly data of bull and bear markets there are used lag lengths of 3 and 4. Statistically significant values, which p-value is lower than 5% are highlighted.

Case		α(%)	β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}	inf. rat	R^2
CAPM	Daily	**0,051**	**-0,336**				0,732	0,096
		(3,44)	(-17,20)					
	Monthly	**0,737**	-0,039				0,562	0,002
		(2,96)	(-0,87)					
3-fact	Daily	**0,051**	**-0,328**	0,047	0,009		0,730	0,097
		(3,41)	(-15,43)	(1,59)	(0,41)			
	Monthly	**0,697**	-0,027	**-0,158**	0,017		0,532	0,016
		(2,82)	(-0,55)	(-2,10)	(0,23)			
4-fact	Daily	**0,041**	**-0,274**	**0,091**	0,018	**0,216**	0,589	0,123
		(2,86)	(-14,16)	(3,12)	(0,78)	(8,59)		
	Monthly	**0,491**	0,040	-0,099	0,023	**0,192**	0,376	0,050
		(2,07)	(0,73)	(-1,19)	(0,30)	(3,38)		
4-fact	Daily	0,047	**-0,180**	**0,160**	0,002	-0,003	0,593	0,042
bull		(1,91)	(-4,71)	(3,35)	(-0,06)	(-0,05)		
	Monthly	0,629	-0,003	-0,105	-0,132	0,120	0,517	0,020
		(1,251)	(-0,027)	(-0,633)	(-0,735)	(1,029)		
4-fact	Daily	0,028	**-0,260**	0,141	-0,028	**0,322**	0,162	0,277
bear		(0,36)	(-4,05)	(1,04)	(-0,37)	(4,20)		
	Monthly	0,0235	0,219	0,254	0,202	**0,394**	0,011	0,128
		(0,016)	(1,054)	(1,00)	(0,65)	(2,11)		

Unexplained positive returns are in line with previous research (Frazzini & Pedersen 2014). BAB factor has negative exposure to market factor in all broad sample models, except in four factor model with monthly returns, where the positive exposure is not statistically significant. BAB factor has statistically significant positive exposure to momentum factor with both data point frequencies in broad and bear market sample. During the bull markets there is basically no exposure to momentum factor, but in bear market times exposure rises to very high level. BAB factor has no statistically significant exposure to value factor. Exposure to SMB factor is inconsistent with different data frequencies. With daily data exposure is positive and with monthly data exposure is negative in every model where SMB is inherent except bear markets model. Explaining power of the model is very limited especially with monthly data. In bear markets with monthly data explaining power rises substantially to 12,8%. Also with daily data explaining power of model rises in bear market times to highest level 27,7%. With both data frequencies it can be seen that three factor model doesn't really add much to CAPM's explaining power, but adding momentum rises explaining power much more.

4.3. Conditional regressions

4.3.1. Compiling alphas and betas

Table 5 shows that even after conditional regression by using smoothened Kalman filter there still exists statistically significant alpha in broad sample, which cannot be explained through CAPM, three factor model or four factor model. Comparing alphas for daily data from table IV and V shows, that conditional factor models capture more of returns related to BAB factor, even though they are unable to capture it completely. Conditional CAPM alpha is much smaller than alpha in unconditional three factor model.

Table 5. Alphas of conditional regressions for BAB factor, July 1990 to December 2015

Table reports means, skewness and kurtosis and information ratios of conditional alphas. Alphas are calculated from equation 17. In parentheses are t-statistics. Statistics are separately presented for CAPM, Fama-French three factor model and four factor model with momentum added. Information ratio equals alphas' mean divided by alphas standard error using annualized values. For 4-factor model there is separated regressions for bull and bear market. Bull (Bear) markets are times when past 12-month return has been over (under) 20% (-20%).Regressions are generated from daily data. Statistically significant values, which p-value is lower than 5% are highlighted.

Case		mean(%)	skewness	kurtosis	inf. rat.
CAPM	Daily	**0,045**	**3,418**	**77,369**	0,655
		(3,12)	(113,85)	(1288,66)	
3-factor	Daily	**0,041**	**3,467**	**80,309**	0,610
		(2,92)	(115,48)	(1337,63)	
4-factor	Daily	**0,032**	**1,350**	**28,283**	0,526
		(2,55)	(44,96)	(471,11)	
4-factor	Daily	0,036	**0,265**	**13,407**	0,590
bull		(1,55)	(4,833)	(122,29)	
4-factor	Daily	-0,012	**1,209**	**22,84**	-0,145
bear		(-0,22)	(11,84)	(112,04)	

Alphas and alphas t-values diminishes substantially from specific unconditional model to related conditional model. By comparing different cases in table 5 it can be seen that adding factors decreases the level of alpha, specially adding the momentum factor. The higher absolute value in bull markets than in broad sample is not statistically significant because of the smaller sample size. Four factor modeling severely decreases alphas skewness and kurtosis. Alpha's skewness and kurtosis also decrease substantially in bull markets.

Table 6 presents means of conditional factor exposure estimates of BAB and means of standard errors of those factor exposure estimates. Same pattern as with unconditional models is observed. Being that BAB factors negative exposure to MKT factor decreases when there is added more explaining factors. Especially on bull markets with four factor model exposure to market factor reaches near zero. In bear markets negative exposure to market factor rises substantially. Beta for size factor doubles from three factor to four factor model. Betas for value factor are really small in all models. Exposure to momentum factor rises remarkably from broad sample and bull sample to bear sample. Significance of factor exposures in the whole time frame cannot be estimated in this context, because distribution for each factor exposure for each time step is unique.

Table 6. Betas of conditional factor regressions for BAB , July 1990 to December 2015

There are presented means of estimated betas and means of estimated standard errors in parentheses. Individual beta estimates and covariance matrices for each time step are achieved by using backward recursive procedure of Kalman smoothing, as in equation 16. Standard errors are defined from singular value decomposition of covariance matrix. Means conditional betas are separately presented for CAPM, Fama-French three factor model and four factor model with momentum added. For 4-factor model there is separated regressions for bull and bear market. Bull (Bear) markets are times when past 12-month return has been over (under) 20% (-20%). For estimating betas for bull and bear market times it is used whole time interval information set. Regressions are generated from daily data.

Case		β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}
CAPM	Daily	-0,259			
		(0,131)			
3-factor	Daily	-0,217	0,076	-0,011	
		(0,133)	(0,113)	(0,082)	
4-factor	Daily	-0,134	0,154	0,018	0,067
		(0,104)	(0,169)	(0,062)	(0,371)
4-factor	Daily	-0,096	0,195	-0,018	-0,060
bull		(0,107)	(0,174)	(0,065)	(0,396)
4-factor	Daily	-0,206	0,185	-0,018	0,326
bear		(0,089)	(0,146)	(0,056)	(0,270)

By comparing means of standard errors it can be seen that time step specific betas can be estimated most accurately for value factor, because standard deviations are the smallest. Momentum exposures are estimated most inaccurately. Comparing time varying beta estimates of with three different samples it can be seen that in bear market times time varying beta estimates are most accurate, compared to broad and bull samples.

4.3.2 Time varying betas

Figure 4 shows time varying market beta estimate, when BAB returns are modelled with conditional CAMP. Figure also shows 95% confidence interval for beta estimate. Beta varies in range from -0,15 to -1,1. For most of the time beta is negative, peaking strongly negative in the end of the year 2011. This is mostly due the extreme observation in BAB returns. It reached it maximum being 31,0% for one day in November 2011. Just 10,2% of the time beta estimate is over the zero level and whole 95% confidence interval is never completely over the zero level.

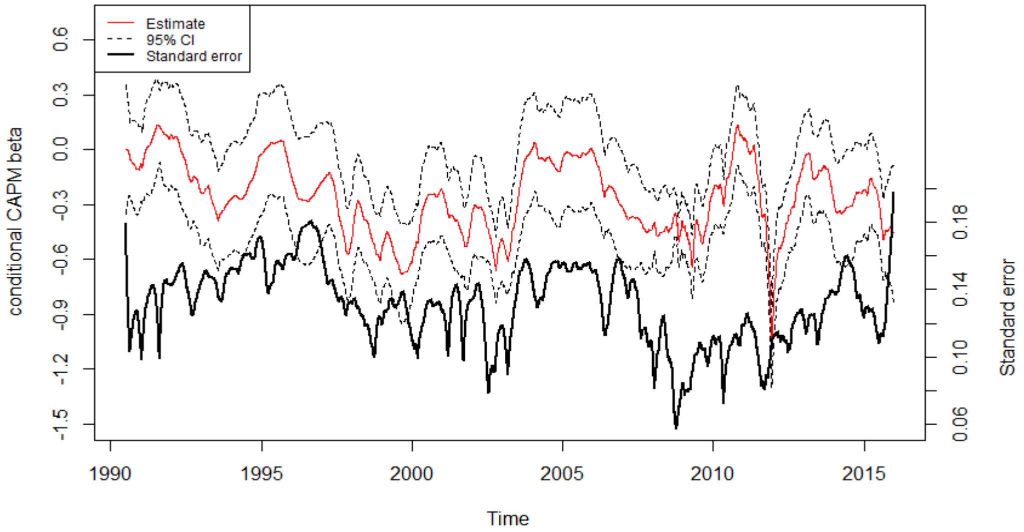


Figure 4. Time varying CAMP beta, July 1990 to December 2015. Figure displays conditional beta estimate, estimate's 95% confidence interval and standard error, when BAB is modelled with CAMP. Beta estimates for each time step are achieved by using backward recursive procedure of Kalman smoothing, as in equation 16 and standard errors from singular value decomposition of covariance matrix.

Actually 46,3% of the time whole confidence interval lies below the zero level. Standard errors of beta estimates tend to go higher in times when exposure moves closer to zero, this leading confidence interval being wider in those times. Higher values of standard deviation in the beginning and the end of time series are due the recursive nature of Kalman smoothing.

Figure 5 presents BAB factor’s time varying exposures to traditional Fama-French three factors. By comparing to figure 4, it can be seen that overall shape of market factor beta line is almost unaffected of adding size and value factor. There is though noticeable increase in the market exposure, but still beta estimates rarely (19,6%) rise over the zero level. BAB has highest exposure to size factor. It can be seen that exposures to size factor and value factor are much less volatile than exposure to market factor. Specially exposure to value factor is pretty stable and near the zero. 52,2% of the time estimated beta for value factor is above the zero level. 68,8% of the time estimated beta to SMB factor is positive.

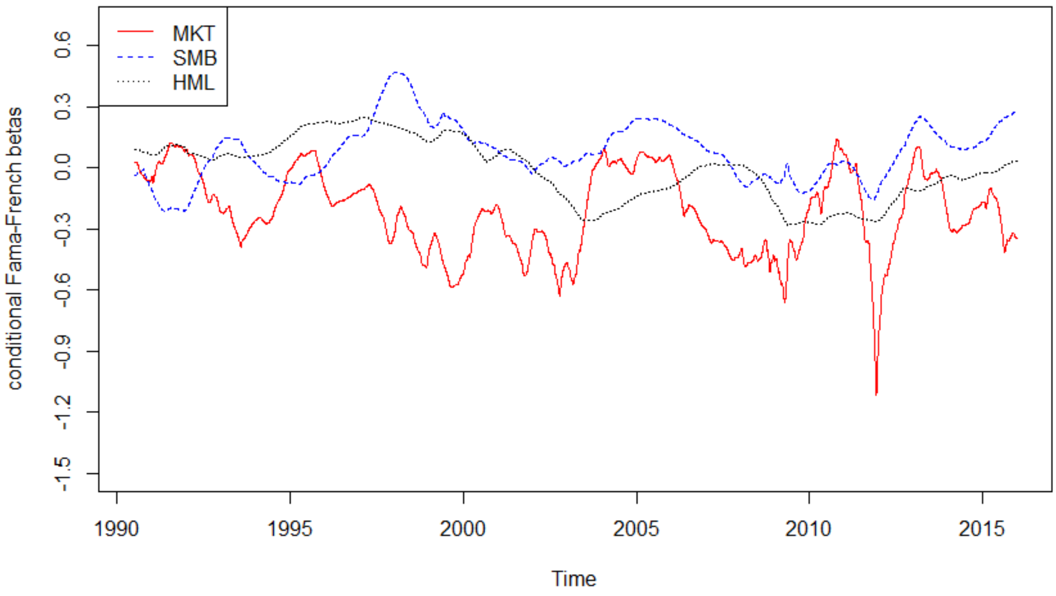


Figure 5. Time varying Fama-French betas, July 1990 to December 2015. Figure displays conditional betas, when BAB returns is regressed on three factor model . Individual beta estimates and covariance matrices for each time step are achieved by using backward recursive procedure of Kalman smoothing, as in equation 16.

Figure 6 shows exposure to market factor with 95% confidence intervals and estimates standard error. Even with three factor model the whole confidence interval for market factor never rises over zero level. Whole confidence interval is below the zero 39,8% (46,3% with CAPM) of the time. These numbers also indicating overall rise of the market factor exposure closer to the zero compared to CAPM modeling. By comparing figures 6, 7 and 8 graphs for estimates and error it can be seen that beta estimate and the accuracy of the estimate for MKT factor is much more time varying than for size or value factor.

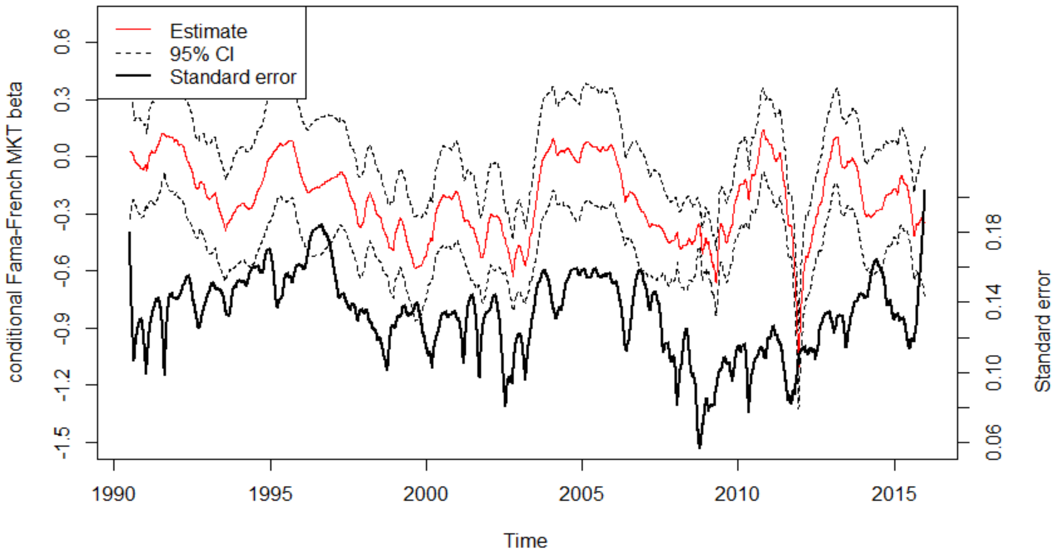


Figure 6. Time varying Fama-French MKT beta, July 1990 to December 2015. Figure displays conditional MKT beta estimate, estimate's 95% confidence interval and standard error of estimate, when BAB is modelled with Fama-French three factor model. Beta estimates for each time step are achieved by using backward recursive procedure of Kalman smoothing, as in equation 15 and standard errors from singular value decomposition of covariance matrix of Kalman smoothing.

Figure 7 shows estimated and 95% confidence interval for beta of size factor in 3-factor model. 12,0% of the time whole confidence interval stands over the zero level and only 2,7% of the time it lies below the zero level. Accuracy of SMB beta estimate is less time varying than accuracy of MKT beta.

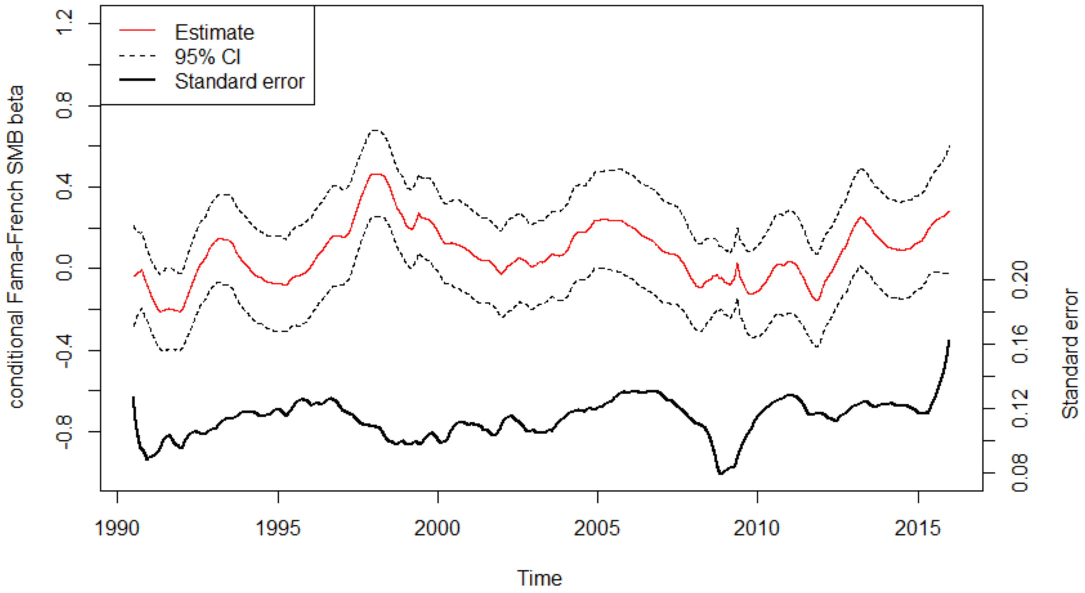


Figure 7. Time varying Fama-French SMB beta, July 1990 to December 2015. Figure displays conditional SMB beta estimates and 95% confidence interval when BAB is modelled with Fama-French three factor model. Beta estimates for each time step are achieved by using backward recursive procedure of Kalman smoothing, as in equation 16 and standard errors from singular value decomposition of covariance matrix of Kalman smoothing.

Figure 8 presents time-varying estimate and 95% confidence interval for exposure value factor. 19,2% of the time the whole confidence interval lies above and 20,9% of time under the zero level of beta. It can be seen that accuracy of HML beta estimate is less time varying than accuracy of MKT or SMB beta.

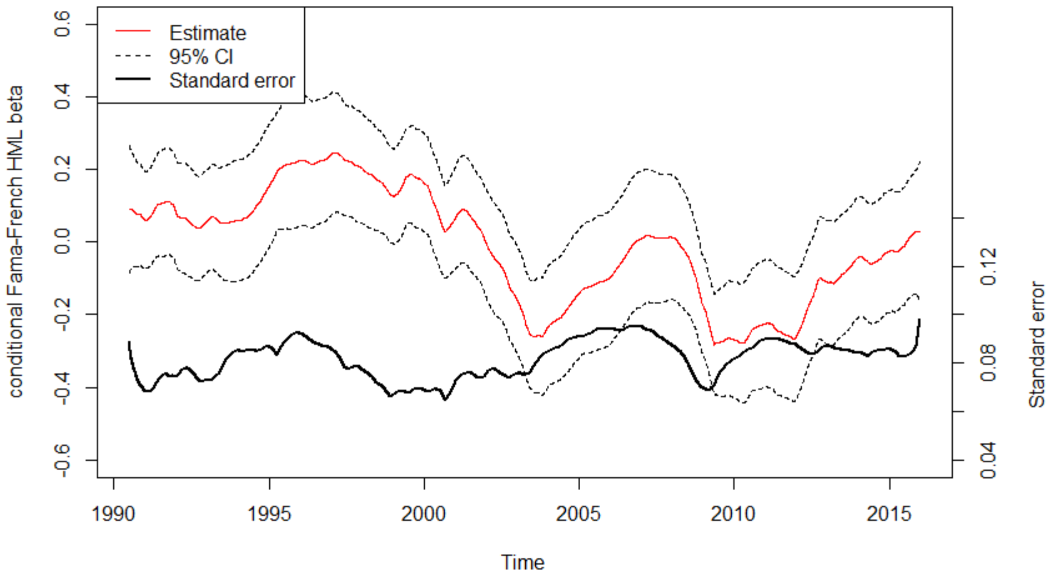


Figure 8. Time varying Fama-French HML beta, July 1990 to December 2015. Figure displays conditional SMB beta estimates and 95% confidence interval when BAB is modelled with Fama-French three factor model. Beta estimates for each time step are achieved by using backward recursive procedure of Kalman smoothing, as in equation 16 and standard errors from singular value decomposition of covariance matrix of Kalman smoothing.

In figure 9 are plotted conditional betas for market, size and value factor in four factor model. Conditional beta for momentum is plotted separately in figure 10, because of it high variation. By comparing figures 4, 5 and 9 it is observed that adding momentum factor cuts down the negative exposure for market factor and also the variation of it. Especially it is observed that the most negative estimates for exposure at the end of 2011 is diminished substantially. 25,5% of the time estimated exposure to market factor is positive, this being higher than with three factor model. This is also indicating that over all exposure closer to zero. By comparing figure 9 and 5 it can be seen that size exposure in four factor model is much more volatile than in three factor model. Also size factor captures most of the extreme values of BAB factor in spring 2009. Estimated exposure to size factor is positive 80,1% of time in the four factor model. This indicating that exposure to size factor is higher level in four factor model than in three factor model. For the value factor estimated exposure is positive 54,1% of the time. Beta of value factor varies around zero even more closely than in three factor model. From comparing figures 9 and 5 it can be seen that variation of exposure to value factor is even smaller in four factor model than in three factor model.

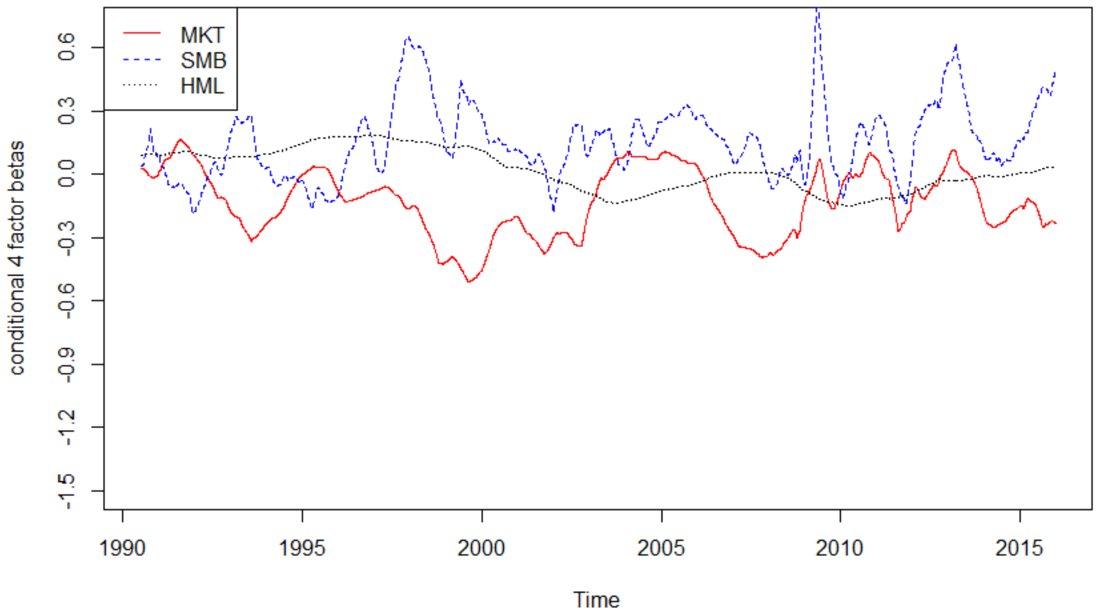


Figure 9. Time varying four factor betas, July 1990 to December 2015. Figure displays conditional betas for MKT, SMB and HML, when BAB returns is regressed on four factor model. Individual beta estimates and covariance matrices for each time step are achieved by using backward recursive procedure of Kalman smoothing, as in equation 16.

From the figure 10 observed huge variation in the momentum factor. Momentum also catches most of the extreme observation in BAB factor returns in the end of year 2011. Momentum beta is 57,7% of the time over zero level. There can be also spotted facts that were present in table VI that in bear market times, techno crash 2000-, financial crises 2008-2009 and European dept crisis 2011-2012, BAB factor's exposure to UMD factor is in the higher level.

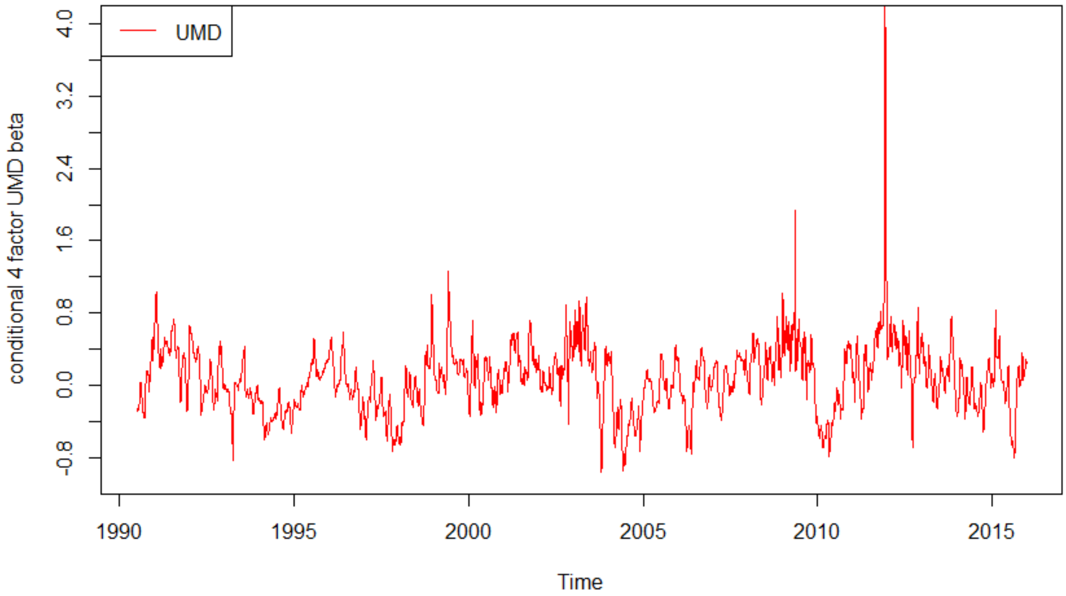


Figure 10. Time varying four factor UMD beta, July 1990 to December 2015. Figure displays conditional beta for UMD, when BAB returns is regressed on four factor model. Individual beta estimates and covariance matrices for each time step are achieved by using backward recursive procedure of Kalman smoothing, as in equation 15.

Figure 11 represents estimate and 95% confidence interval of market beta in four factor model. Here it is observed that 34,7% of the time whole confidence interval is below zero level and never completely above it. This confirming the fact, which was present in table VI, that BAB exposure to marker factor is less negative, when factors are added. Comparing standard error graphs of figures 11, 6 and 4 confirms the fact that accuracy of MKT beta estimates is better and less time varying with four factor model than with other models. Also differences between bull and bear market conditions can be seen. After techno bubble crash in year 2000 and liquidity crash in year 2008 it can be seen that BAB factor’s exposure the MKT factor is lower level.

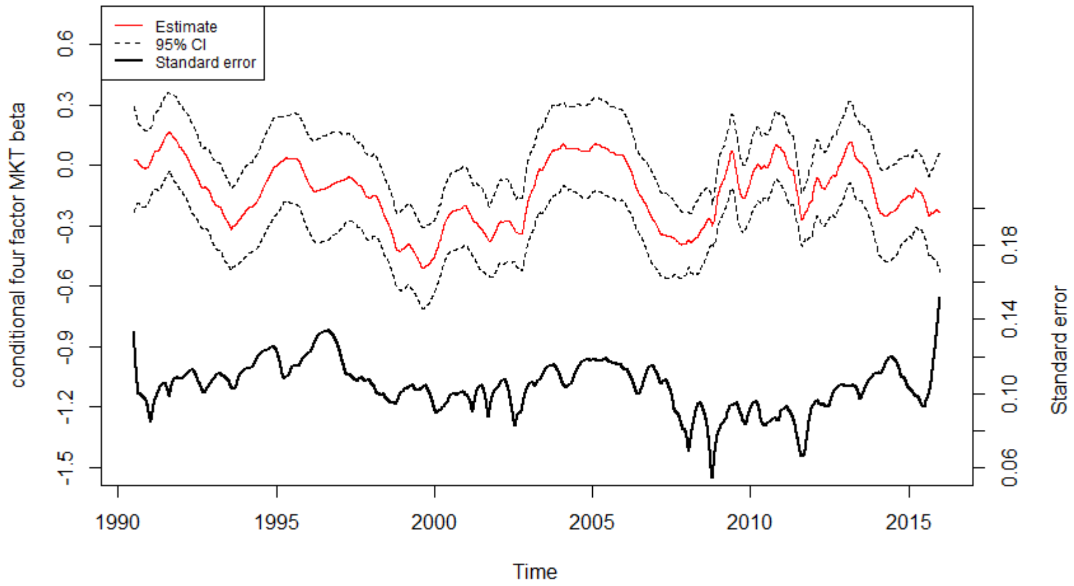


Figure 11. Time varying four factor MKT beta, July 1990 to December 2015. Figure displays conditional MKT beta estimate, estimate's 95% confidence interval and standard error when BAB is modelled with four factor model. Beta estimates for each time step are achieved by using backward recursive procedure of Kalman smoothing, as in equation 16 and standard errors from singular value decomposition of covariance matrix of Kalman smoothing.

Figure 12 shows BAB factor returns time varying exposures to size factor under four factor model. 14,6% of the time whole confidence interval of value factor beta is over the zero level and never (0%) under it. These percentages together with percentages received from figure 7 (12,0%, 2,7%) being in line with table VI, suggesting BAB to have higher exposure to SMB when UMD is added to explanatory variables. It can be seen that variance of estimated SMB betas rises from three factor model to four factor model and variance is also less stable. Contrary to three factor model in the four factor model SMB beta estimate's variance is more time varying than the MKT beta estimate's variance.

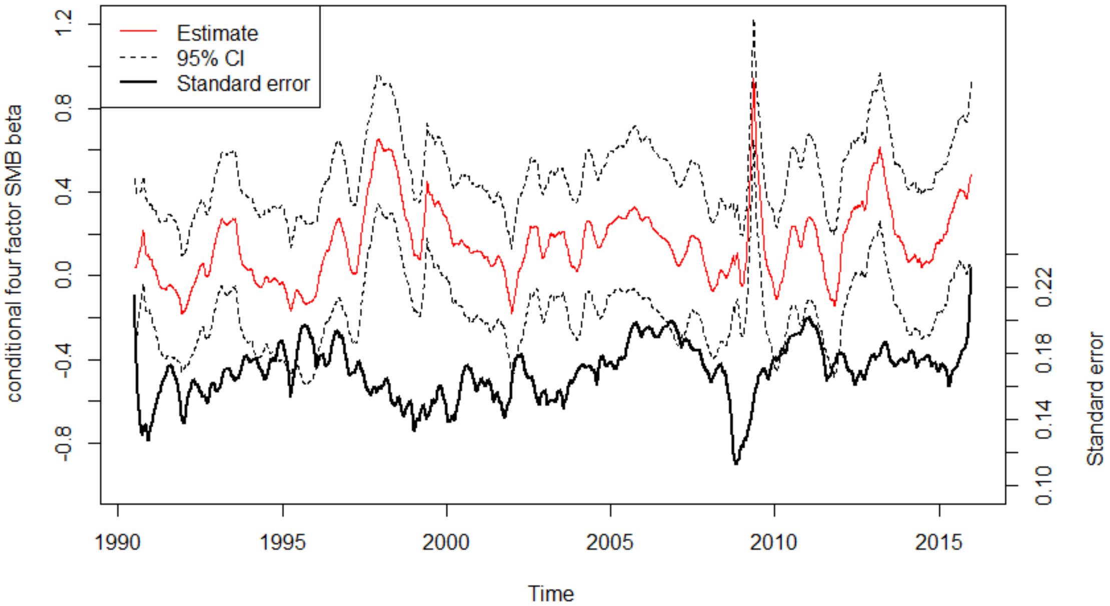


Figure 12. Time varying four factor SMB beta, July 1990 to December 2015. Figure displays conditional SMB beta estimate, estimate's 95% confidence interval and standard error, when BAB is regressed on four factor model. Beta estimates for each time step are achieved by using backward recursive procedure of Kalman smoothing, as in equation 16 and standard errors from singular value decomposition of covariance matrix of Kalman smoothing.

Figure 13 shows conditional betas for value factor and betas 95% confidence interval. Whole confidence interval is above the zero level 21,2% of the time and completely below 8,7% of the time. These numbers together with same percentages from figure 6 (19,2%; 20,9) confirming the fact already present in table VI. That BAB exposure to HML is higher level in four factor model than in 3-factor model. Variance of the HML beta estimate decreases little from three factor model to four factor model and also variance is more stable.

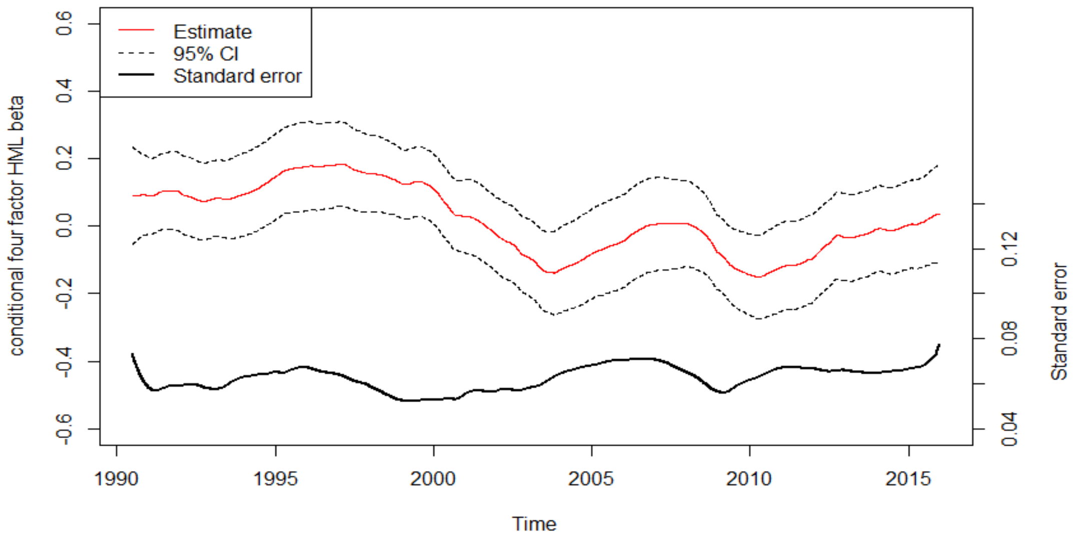


Figure 13. Time varying four factor HML beta, July 1990 to December 2015. Figure displays conditional HML beta estimate, estimate's 95% confidence interval and standard error, when BAB is regressed on four factor model. Beta estimates for each time step are achieved by using backward recursive procedure of Kalman smoothing, as in equation 16 and standard errors from singular value decomposition of covariance matrix of Kalman smoothing.

Figure 14 presents conditional beta estimates for momentum factor and estimates 95% confidence interval. It is observed huge variation in estimates and wider confidence intervals compared to other factor betas. Only 6,2% of time confidence interval is completely over the zero level of beta. 0,7% of the time confidence interval stands completely under the zero level of beta. It can be seen that accuracy of beta estimates is highly time varying compared to any other factor. Also figure states that in crisis 2000-, 2008-2009 and 2012 beta estimates accuracy is better, as standard error of the estimate decreases.

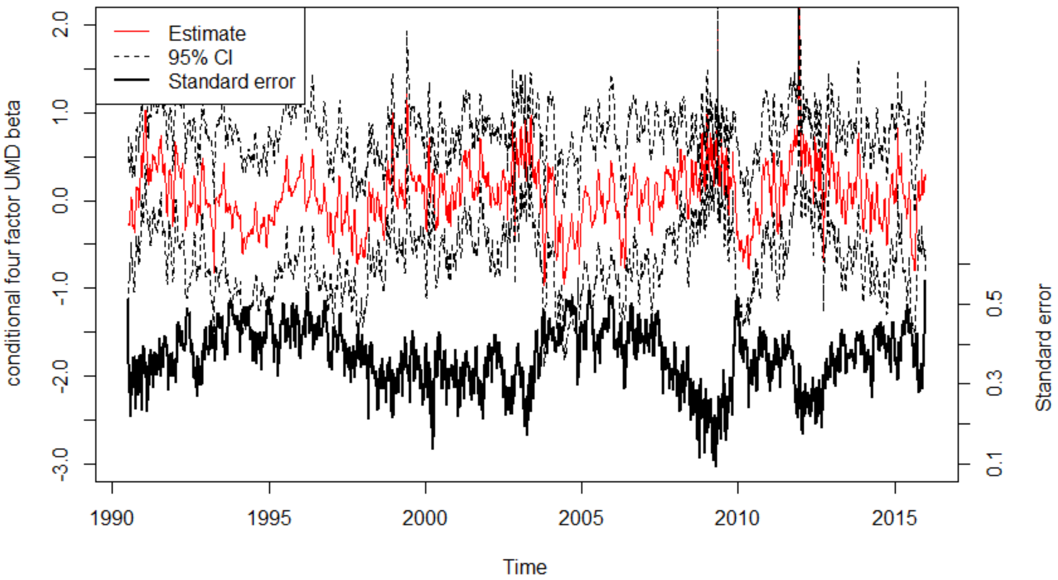


Figure 14. Time varying four factor UMD beta, July 1990 to December 2015. Figure displays conditional UMD beta estimate, estimate's 95% confidence interval and standard error, when BAB is regressed on four factor model. Beta estimates for each time step are achieved by using backward recursive procedure of Kalman smoothing, as in equation 16 and standard errors from singular value decomposition of covariance matrix of Kalman smoothing.

4.4. Rolling regressions

Time varying returns of BAB factor are evaluated by 5 year rolling regression. It is checked can real life investor achieve returns, which are unobtainable through other factors, in 5 year time interval by tilting his portfolio towards BAB factor.

Figure 15 shows t-values of alphas under 5 year rolling regression by 4 factor model. Figure also describes cumulative log return of the market. It is used log returns for market returns to make easier to compare different times market reactions. It is noticed that t-value of alpha never gets statistically significant negative values. It reaches it minimum after techno bubble crash in the beginning of the year 2001, but it is still far out of being statistically significant, t-value being -1. T-value reaches it maximum during the boom before the financial crisis. During the crisis t-value starts to melt down and after that t-value stays positive, but never reaches statistically significant level. 25,5% of time alpha is statistically significant and t-value over 1,96. 90,0% of time alpha is positive.

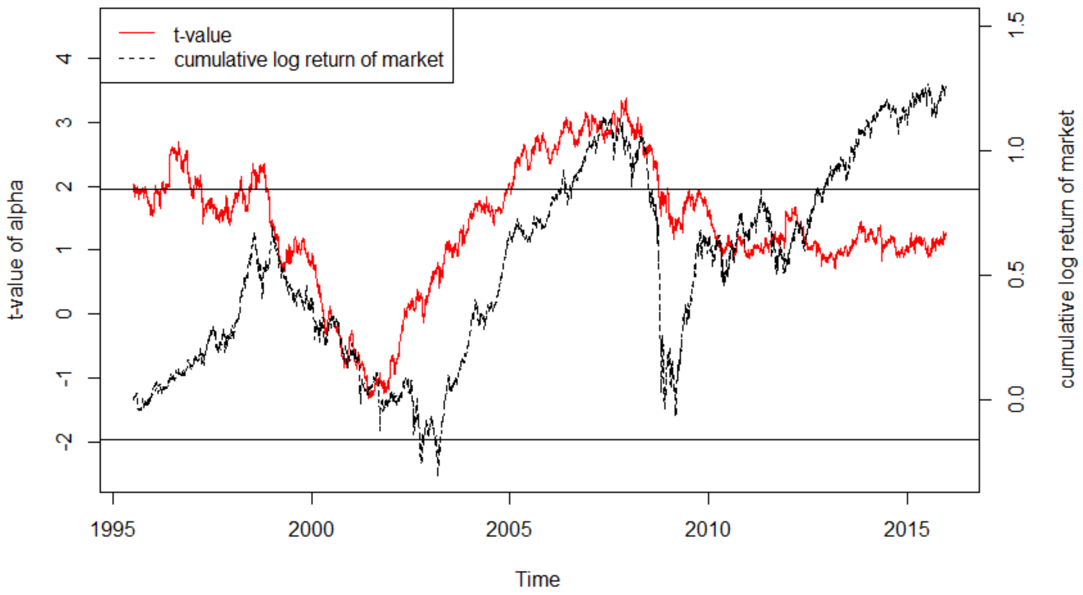


Figure 15. Rolling regression 4-factor alpha's t-values for BAB and cumulative log return of market, July 1995 to December 2015. Figure displays 5 year rolling regression alpha's t-values, when BAB returns are regressed on four factor model with daily data, as in equation 18. There is used Newey-West method with lag length seven to take into account heteroscedasticity and autocorrelation in regressions error terms. Figure also shows cumulative log return of MKT factor with daily data.

Figures presenting 5 year rolling alpha t-values under CAPM and 3 factor model can be found from appendix 2 and 3. Graphs look pretty much the same, but the overall alphas t-values move little higher level when there is less explaining factors. With CAPM 29,4% of time t-value is over 1,96 and alpha statistically significant and positive. 92,8% of time t-value is positive, meaning that alpha is positive. With 3 factor model 27,3% of the time alpha is statistically significantly positive and 91,9% of time alpha is positive. Comparing appendixes 2 and 3 it can be seen that size and value factors can't really capture much of the 5 year rolling alpha. Comparing appendix 3 and figure 13 shows that momentum can't do much better in capturing alpha.

Figure 16 shows five year rolling window estimate for market beta, when BAB factor is regressed on four factor model. Figure also presents 95% confidence interval for estimate and standard deviation. BAB factors estimated exposure to market factor is negative in whole time interval and statistically significant 97,0% of the time. Figure is confirming same fact with the figures 9 and 11. That BAB factor has most of the time negative market exposure.

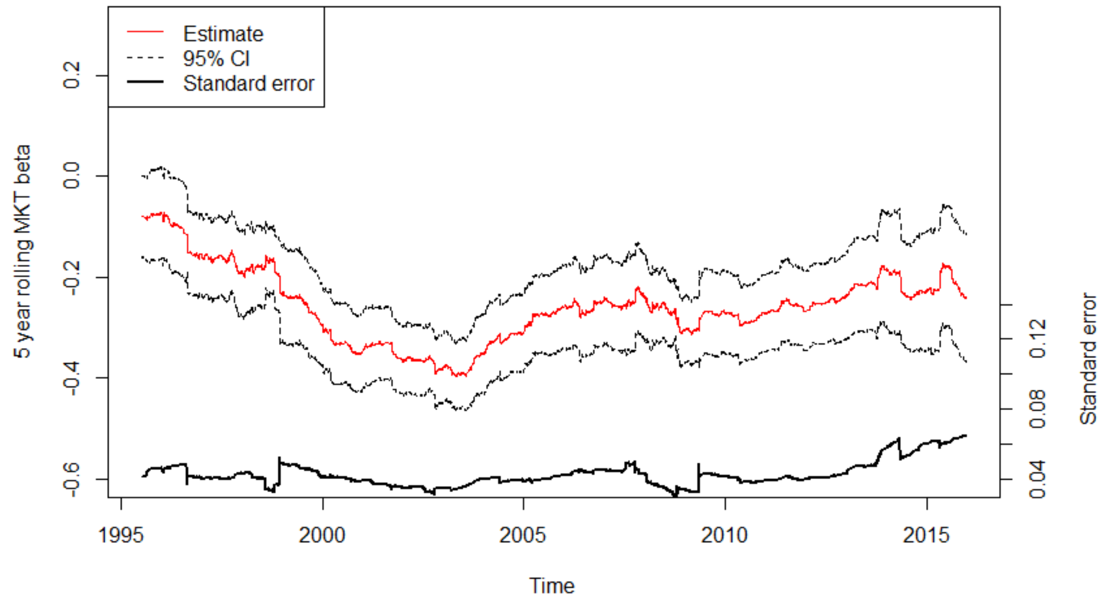


Figure 16. Rolling regression 4-factor MKT beta, July 1995 to December 2015. Figure displays 5 year rolling regression market beta estimates, 95% confidence interval and standard error, when BAB returns are regressed on four factor model. There is used Newey-West method with lag 7 to take into account heteroscedasticity and autocorrelation in regressions error terms.

Figure 17 shows rolling window estimate for size beta, when BAB factor is regressed on four factor model. There are also presented 95% confidence interval for estimate and standard deviation. In May 2009 there is huge increase in the exposure, at the same time there are spotted extreme observations in BAB factor return. Decrease in the factor exposure in May 2015 is due the fact that extreme observation drop out of the 5 year rolling window. BAB factors estimated exposure to size factor is positive 93,0% of the time and positively statistically significant 43,3% of the time. Only 0,3% of the time exposure to size factor is negative and statistically significant. Figure 17 illustrates same phenomenon as figures 9 and 12. BAB factor is mostly exposed positively to size factor.

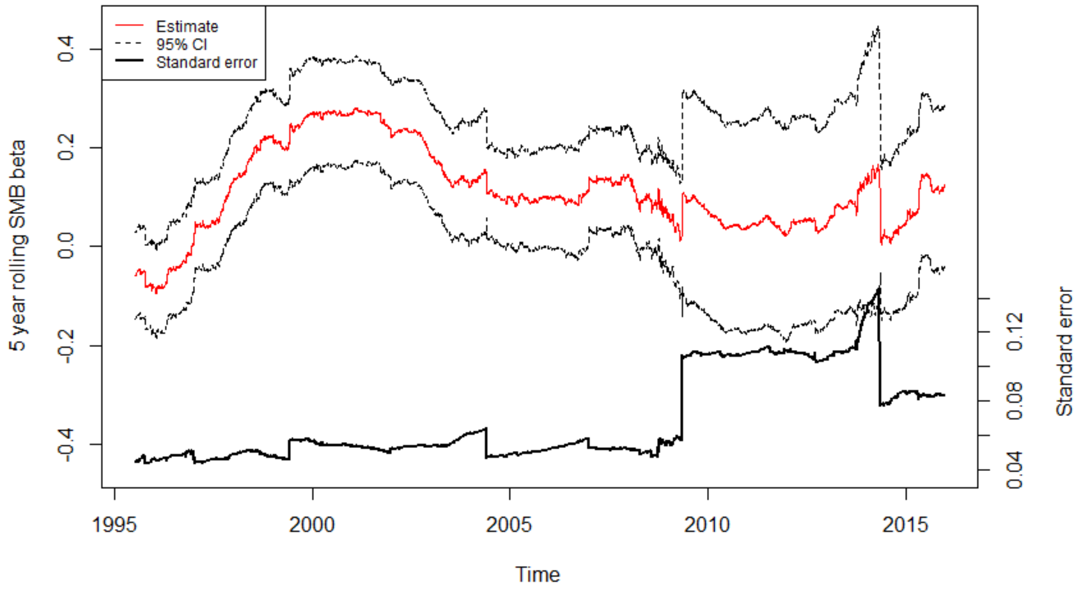


Figure 17. Rolling regression 4-factor SMB beta, July 1995 to December 2015. Figure displays 5 year rolling regression size beta estimates, 95% confidence interval and standard error, when BAB returns are regressed on four factor model. There is used Newey-West method with lag seven to take into account heteroscedasticity and autocorrelation in regressions error terms.

Figure 18 shows rolling window estimate for value beta, when BAB factor is regressed on four factor model. There are also presented 95% confidence interval for estimate and standard deviation. Sudden drop in the value exposure is observed, at same time as the peak in the size exposure, but with smaller scale. BAB factors estimated exposure to value factor is negative 59,4% of the time and negatively statistically significant 23,4% of the time. 34,9% of the time exposure to size factor is positive and statistically significant.

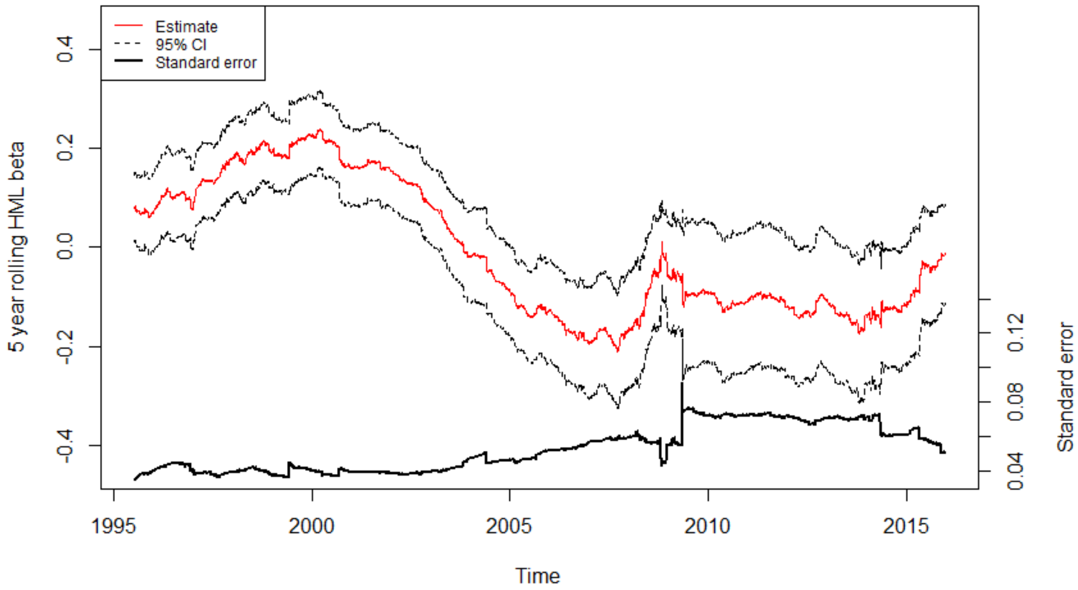


Figure 18. 4-factor rolling regression HML beta, July 1995 to December 2015. Figure displays 5 year rolling regression size beta estimates, 95% confidence interval and standard error, when BAB returns are regressed on four factor model. There is used Newey-West method with lag seven to take into account heteroscedasticity and autocorrelation in regressions error terms.

Figure 19 illustrates same as previous figures, but now for momentum factor. Once again there is spotted sudden peaks and drops due the extreme observations in BAB factor returns. BAB factors estimated exposure to momentum factor is positive 87,3% of the time and positively statistically significant 77,4% of the time. 6,6% of the time exposure is negative and statistically significant.

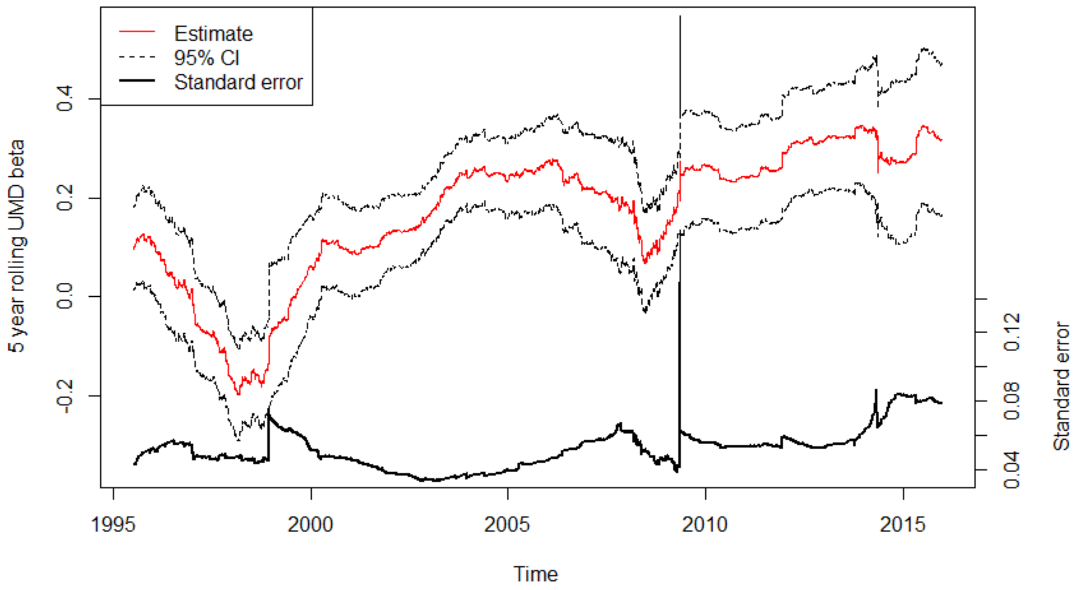


Figure 19. 4-factor rolling regression UMD beta, July 1995 to December 2015. Figure displays 5 year rolling regression size beta estimates, 95% confidence interval and standard error, when BAB returns are regressed on four factor model. There is used Newey-West method with lag seven to take into account heteroscedasticity and autocorrelation in regressions error terms.

5 CONCLUSIONS

Static regression models are not able the capture returns related to BAB factor. Unconditional CAPM risk adjusted BAB alpha is even higher than raw returns. In daily basis even three factor risk adjusted BAB alpha exceeds raw returns. Static Fama-French three factor model can't really capture returns related to BAB factor. Conditional models do better, but even with them there exists statistically significant alpha. Four factor model captures more of BAB factor returns than other models, because BAB factor is positively and closely related to momentum factor. This can be seen from every regression where UMD factor is present. There are high betas related to momentum factor and comparing three factor model and four factor shows that adding only momentum severely decreases alpha. BAB factor returns are positively skewed unlike it would be anticipated from factor which produces statistically significant positive alpha. Under cumulative prospect theory investors in their utility function overweight fat tails of probability distribution (Tversky & Kahneman 1992) of returns. This should cause positively skewed securities to become overpriced, but returns generated by the BAB factor doesn't seems to confirm this fact. Bull and bear market analysis gives indication that BAB factor success is decreased markedly during bear markets, times wen also liquidity is more constrained.

BAB returns correlations with MKT, HML and UMD returns moves further away from zero during bear markets. Correlations that are inherent in normal market conditions get stronger during bear markets, even the negative ones. Exceptionally high negative correlation with market is noteworthy. Contrary to previous, BAB correlation with SMB is lower in bear markets than in broad sample.

Time varying beta survey suggests that BAB factor has never suffered of statistically significant positive market exposure. BAB returns analyze under conditional four factor model suggests that over one third of the time exposure to market has been statistically significant and negative. Suggesting that investing to BAB factor and to market offers great divarication benefits. Actually these benefits compared of just investing to BAB factor are mostly vanished through negative skewness and lower mean of the market return. This is shown in-sample backtesting in Appendix 3 by investing 50/50 portfolio of BAB and MKT factor with monthly rebalancing. It is used log returns to achieve better scalability of different states market movements. During techno boom 50/50 portfolios cumulative return raised temporarily.

Momentum and BAB connection in factor level could be related to aggregate market tendencies. In the aggregate market level high volatility, low prices and low ex-post returns are known to be related and vice versa. These tendencies occur in the individual stock level. Stocks with high ex-post returns, which are also stocks that have long position in momentum factor, tend to lower betas than stocks with low ex-post returns. It has been found that momentum factor also has statistically significant negative market exposure. (Daniel & Moskowitz 2013.) These findings may lead to the fact that there are similarities between the constituents of momentum and BAB factor. Huge variation in the momentum beta can be related to the fact that UMD factor constituents are rebalanced monthly as other explaining dynamic factors SMB and HML are rebalanced yearly. Investor who his harvesting higher returns by tilting his portfolio to momentum factor should be causes of tilting his portfolio towards BAB factor, because correlation between these two increases exceptionally in bear market times. Both factors seems to provide same kind diversification benefits for bad times. BAB factor correlation though stays negative with MKT factor in bull markets too.

Rolling regressions show that in the five years interval BAB rarely generates statistically significant positive alpha. From rolling regressions it can be seen that in times when aggregate market goes down also risk adjusted BAB returns t-value tend to go lower level, confirming facts that were presented in unconditional and conditional regressions for bear markets. But even in the bad times BAB factor risk adjusted returns tend to be positive or at least negative without statistical significance. In whole time interval rolling regression alphas don’t get statistically significant negative values. Tilting portfolio towards BAB factor does not really penalize investor in bad times by unique way, not the way that wouldn’t be captured by other factors.

Four factor rolling regression confirms findings of time varying betas, which were present in Kalman filter models. BAB factors exposure to market factor is mostly negative. Actually it is negative in whole time interval in rolling regression. Also BAB factors positive exposure to size factor for most of the time inherent in Kalman modeling is confirmed by the rolling regression. Exposure to value factor in the rolling regression wanders both sides of zero as with Kalman filter modeling. Rolling regression suggests more strongly BAB factors positive exposure to momentum factor than Kalman filter model.

There has been statements that BAB returns are actually generated from taking tilts towards stable industries and actually high returns rise from value tilt. This criticism conveys the idea that those high returns of BAB factor are result of path-dependent data mining and this makes tilting towards BAB being extremely dangerous. There is though evidence that BAB factor earns extremely strong risk adjusted returns within-industry level. Within-industry BAB factor has generated positive returns in all of 49 industries in US and in 60 of 70 in global level. Also aggregate industry neutral BAB factor, generated by putting those within-industry BAB returns together, has high four factor risk adjusted returns. Aggregate industry neutral BAB factor actually has smaller value tilt than ordinary BAB.(Asness et al. 2014.)

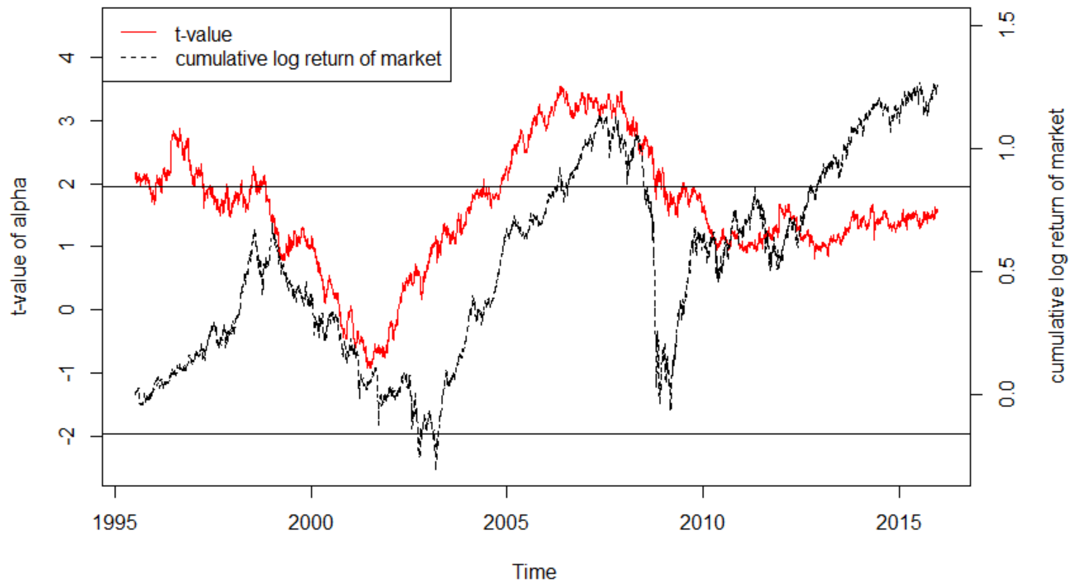
BAB factor creation has been criticized for the fact that in generating factor returns it is used lagged beta and becoming returns. Lagged beta is used as scaling parameter to get equal betas for long and short positions, so that total position would be market neutral. CAPM does not predict linear relationship between the lagged beta and becoming returns. It states that linear relationship occurs between the beta and return in the same time interval. In the BAB factor creation this has been tried to solve by shrinking passed betas towards the mean, but still there exists the problem. Lagged betas are lousy predictors for becoming beta (Blume 1975). If shrinking happens faster ($\gamma < 0{,}6$) than BAB creation model, then BAB factor long position risk adjusted returns are overstated and short position understated. Causing whole factor beta adjusted returns be overstated.

Low beta and low volatility strategies have raised their heads in recent years. There are huge inflows on ETFs, which use either low beta or low volatility strategies. Past success of these strategies has raised a question have the low beta stocks got overpriced. High ex-post returns can be the due the fact too that valuations of low beta stocks have raised to unsustainable level. If there exists high valuation of low beta stocks compared high beta stocks, is the valuation difference going the mean revert and is mean reverting executed through adjustments in prices. Time interval of data collection in this research quiet modest (25 years) in the economic scale to observe extreme scenarios related to BAB returns. High returns and positive skewness of BAB can be due the fact that rare extreme crashes are yet not happened, but those risk of those crashes has been connected to low beta stocks and for this reason they have been in offered huge returns. Also low beta stocks can suffer from strong exposure to some other risk factor, which has not been inherent in researches. This factor could be for example liquidity, even though BAB factor in Belgium didn't suffer in recent liquidity crisis 2008. Especially link between the funding liquidity and BAB should positive because constrained funding liquidity should drive investors towards higher beta stocks.

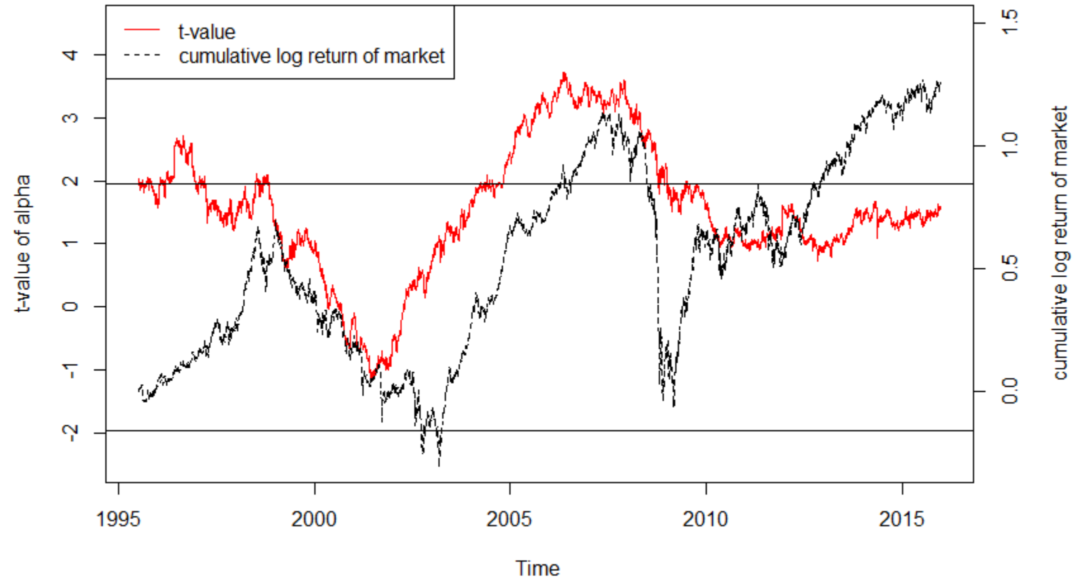
APPENDIX

	Daily n=6654			Monthly n=306		
Factor	Means	Skew.	Kurtosis	Means	Skew.	Kurtosis
MKT	0,028	-0,107	6,504	0,632	-1,005	4,753
SMB	-0,007	-0,045	2,291	-0,149	-0,423	3,532
HML	0,023	0,395	5,42	0,495	0,010	0,298
UMD	0,040	0,026	11,165	0,891	-1,096	6,970

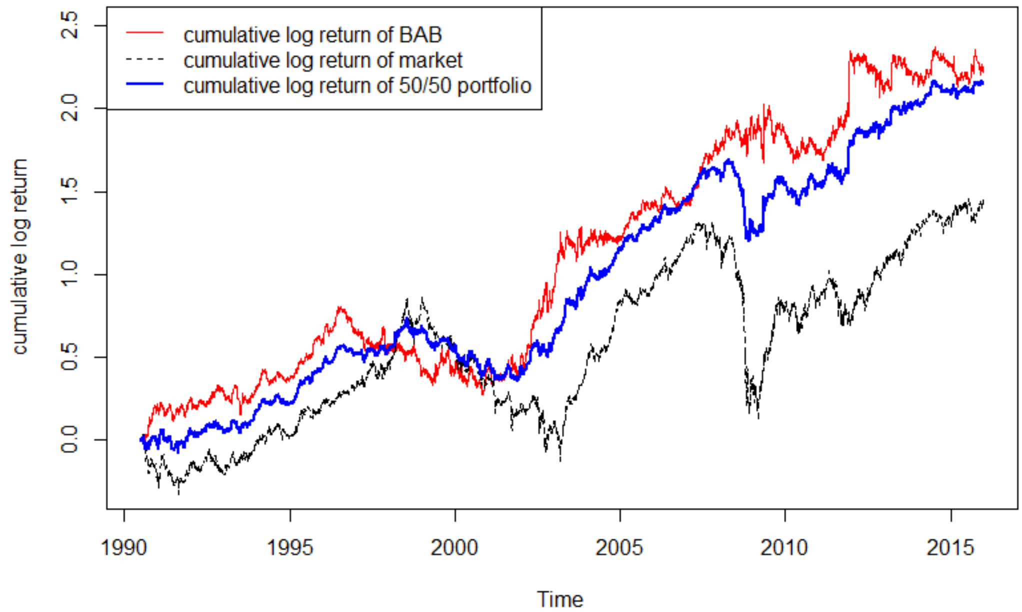
Appendix 1. Explaining factors summary statistics, July 1990 to December 2015. The table reports means, skewnesses and kurtosis of the market, size, value and momentum factor returns.



Appendix 2. Rolling regression CAMP alpha's t-value and cumulative log return of market, July 1995 to December 2015. Figure displays 5 year rolling regression alpha's t-values, when BAB returns are regressed on CAMP. There is used Newey-West method with lag seven to take into account heteroscedasticity and autocorrelation in regressions error terms.



Appendix 3. Rolling regression Fama-French alpha's t-value and cumulative log return of market, July 1995 to December 2015. Figure displays 5 year rolling regression alpha's t-values, when BAB returns are regressed on CAMP. There is used Newey-West method with lag seven to take into account heteroscedasticity and autocorrelation in regressions error terms.



Appendix 4. Cumulative log returns, July 1990 to December 2015. Figure shows cumulative log returns of BAB factor, MKT factor and 50%/50% diversified portfolio between those.

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